Chapter 7 (Confidence Intervals – one population) This chapter presents the beginning of inferential statistics. The two major applications of inferential statistics involve the use of sample data to (1) estimate the value of a population parameter, and (2) test some claim (or hypothesis) about a population.

In this chapter we will look at methods for estimating values of population parameters: such as proportions, means, and variances. As well as methods for determining sample sizes necessary to estimate those parameters.

Definitions to Know: Point estimate, confidence interval, confidence level also called the degree of confidence, and critical value.

A critical value is the number on the borderline separating sample statistics that are likely to occur from those that are unlikely to occur. The number $z_{\alpha/2}$ is a critical value that is a *z* score with the property that it separates an area of $\alpha/2$ in the right tail of the standard normal distribution.

Example 1:

Find $z_{\alpha/2}$ (CRITICAL VALUE) for 95% Degree of Confidence

Find $z_{\alpha/2}$ (CRITICAL VALUE) for 90% Degree of Confidence

Find $z_{\alpha/2}$ (CRITICAL VALUE) for 99% Degree of Confidence

When data from a simple random sample are used to estimate a population proportion *p*, the margin of error, denoted by *E*, is the maximum likely (with probability $1 - \alpha$) difference between the observed proportion $\hat{p}$ and the true value of the population proportion *p*.

7.2 Estimating a Population Proportion Assumptions: The sample is a simple random sample, The conditions for the binomial distribution are satisfied, The normal distribution can be used to approximate the distribution of sample proportions because $np \geq 5$ and $nq \geq 5$ are both satisfied.

Example 2: 829 adult Minnesotans were surveyed, and 51% of them are opposed to the use of the photo-cop for issuing traffic tickets. Use these survey results.

a) Find the margin of error *E* that corresponds to a 95% confidence level. b) Find the 95% confidence interval estimate of the population proportion *p*. c) Based on the results, can we safely conclude that the majority of adult Minnesotans oppose use the photo-cop?

Example 3: Determining Sample Size

Suppose a sociologist wants to determine the current percentage of U.S. households using e-mail. How many households must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points? a) Use this result from an earlier study: In 1997, 16.9% of U.S. households used e-mail (based on data from *The World Almanac and Book of Facts*).

b) Now assume that we have no prior information suggesting a possible value of $\hat{p}$

Finding the Point Estimate and E from a Confidence Interval

Point estimate of $\hat{p}$: $\hat{p} = \frac{(upper\ confidence\ limit\ +\ lower\ confidence\ limit)}{2}$

Margin of Error: $E = \frac{(upper\ confidence\ limit\ -\ lower\ confidence\ limit)}{2}$

Example 4:

a) **Find the Point Estimate for the following confidence interval $5 < P < 10$**

b) **Find the margin of error for the following confidence interval $5 < P < 10$**

7.3 Estimating a Population Mean (σ known) Assumptions: The sample is a simple random sample. The value of the population standard deviation σ is known. Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

A confidence interval is a range (or an interval) of values used to estimate the true value of the population parameter. The confidence level gives us the success rate of the procedure used to construct the confidence interval. **What are some of the population parameters we have worked with?**

The margin of error is the maximum likely difference observed between sample mean $\bar{x}$ and population mean μ,and is denoted by E.

Example 1: A study found the body temperatures of 106 healthy adults. The sample mean was 98.2 degrees and the population standard deviation is known to be 0.62 degrees. Find the margin of error E and the 95% confidence interval for μ.

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Example 2: Assume that we want to estimate the mean IQ score for the population of statistics professors. How many statistics professors must be randomly selected for IQ tests if we want 95% confidence that the sample mean is within 2 IQ points of the population mean? Assume that $\sigma = 15$, as is found in the general population.

Estimating a Population Mean (σ not known) Assumptions: The sample is a simple random sample. The value of the population standard deviation σ is NOT known. Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$. Use the student t distribution.

Degrees of Freedom (*df*) corresponds to the number of sample values that can vary after certain restrictions have been imposed on all data values (this is needed when using the t distribution)

Example 1:

Find $t_{\alpha/2}$ (CRITICAL VALUE) for 95% Degree of Confidence

When n = 10

When n = 20

When n =30

Find $t_{\alpha/2}$ (CRITICAL VALUE) for 90% Degree of Confidence

When n = 10

When n = 20

When n =30

Find $t_{\alpha/2}$ (CRITICAL VALUE) for 99% Degree of Confidence

When n = 10

When n = 20

When n =30

Example 2:

Flesch ease of reading scores for 12 different pages randomly selected from J.K. Rowling's *Harry Potter and the Sorcerer's Stone found a* sample mean of 80.75 and a standard deviation of 4.68

Find the 95% interval estimate of μ, the mean Flesch ease of reading score. (The 12 pages' distribution appears to be bell-shaped.)

The output of the Flesch Reading Ease formula is a number from 0 to 100, with a higher score indicating easier reading. The average document has a Flesch Reading Ease score between 6-70
http://en.wikipedia.org/wiki/FleschKincaid_Readability_Test

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Finding the Point Estimate and E from a Confidence Interval

Point estimate of μ:

$\bar{x} = \frac{(upper\ confidence\ limit + lower\ confidence\ limit)}{2}$

Margin of Error: $E = \frac{(upper\ confidence\ limit - lower\ confidence\ limit)}{2}$

7.4 Estimating a Population Variance Assumptions: The sample is a simple random sample.
The population must have normally distributed values (even if the sample is large). Need the Chi-square distribution.

The chi-square distribution is not symmetric, unlike the normal and Student t distributions.
As the number of degrees of freedom increases, the distribution becomes more symmetric. The values of chi-square can be zero or positive, but they cannot be negative.
The chi-square distribution is different for each number of degrees of freedom, which is *df* = *n* – 1 in this section. As the number increases, the chi-square distribution approaches a normal distribution.

In Table A-4, each critical value of χ^2 corresponds to an area given in the top row of the table, and that area represents the total region located to the right of the critical value.

Example 1:

Find the CRITICAL VALUES for 95% Degree of Confidence

When n = 10

When n = 20

When n =30

EXAMPLE 2: A study found the body temperatures of 106 healthy adults. The sample mean was 98.2 degrees and the sample standard deviation was 0.62 degrees. Find the 95% confidence interval for σ.

Chapter 7 (Confidence Intervals – one population) SUMMARY

Wording	Estimating	Parameter	Formula used	critical value	TI 84 select STAT then TESTS
find a ___% confidence interval for the population mean (σ known)	A Population Mean	μ (σ known)	$\bar{x} - E < \mu < \bar{x} + E$ $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	Table A-2	Z-interval
find a ___% confidence interval for the population mean (σ not known)	A Population Mean	μ (σ unknown)	$\bar{x} - E < \mu < \bar{x} + E$ $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$	Table A-3	T-interval
find a ___% confidence interval for the population standard deviation	A Population Standard Deviation	σ	$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$	Table A-4	--------
find a ___% confidence interval for the population proportion	A Population Proportion	P	$\hat{p} - E < P < \hat{p} + E$ $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$ where $\hat{p} = \frac{x}{n}$ and $\hat{q} = 1 - \hat{p}$	Table A-2	1-propZint

Common Critical values: (only for $z_{\alpha/2}$)

Confidence Intervals	Critical Value
.90	1.645
.95	1.96
.99	2.575
.98	2.33

Sample Size Determination: Find the sample size needed to ……

$n = \frac{(z_{\alpha/2})^2 (0.25)}{E^2}$	Use when σ, $\hat{p}$ and $\hat{q}$ are not given
$n = \frac{(z_{\alpha/2})^2\, \hat{p}\hat{q}}{E^2}$	Use when $\hat{p}$ and $\hat{q}$ are given
$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{E}\right]^2$	Use when σ is given

When finding sample size ALWAYS round up.
Example: n = 134.01 would be n = 135