Chapter 4. Probability and Counting Rules

4-2. Sample Spaces and probability

Probability: is the chance of an event occurring.
e.g. card games, slot machines, lotteries, or stocks.

Probability is the basis of inferential statistics.

Probability experiment: is a chance process that leads to well-defined results called outcomes.

e.g. flipping a coin, rolling a dice, or drawing a card from a deck.

Outcome: is the result of a single trial of a probability experiment.

Sample space: is the set of all possible outcomes of a probability experiment.

Sample spaces can be found by:

i) observation and reasoning
ii) a tree diagram

Tree diagram: is a device consisting of line segments emanating from a starting point and also from the outcome point. It is used to determine all possible outcomes of a probability experiment.

An event: consists of a set of outcomes of a probability experiment.

Simple event: An event with one outcome.
e.g. a die is rolled to get a 6

Compound event: An event with two or more outcomes.
e.g. rolling a die to get an odd number.

Types of probability:

1) **Classical probability, P(E):** assumes that all outcomes in the sample space are equally likely to occur. It uses the sample space.

$$P(E) = \frac{\text{Number of outcome in E}}{\text{Total no. of outcomes in the sample space}}$$

$$= \frac{n(E)}{n(S)}$$ where E is the event and S is sample space.

Equally likely events are events that have the same probability of occurring.
*Probabilities can be expressed as fractions, decimals, or percentages.

Rounding rule for probabilities:
Reduced fractions or two or three decimal places.
When using tables, use the number of decimal places given in table.

Probability rules:

1) The probability of an event E is a number between and including 0 and 1.

$$0 \leq P(E) \leq 1$$

i.e. probability can not be negative or greater than 1.

2) If an event cannot occur, its probability is 0.

3) If an event E is certain, then the probability of E is 1.

4) The sum of the probabilities of the outcomes in the sample space is 1.

Complementary Events: The complement of an event E is the set of outcomes on the sample space that are not included on the outcomes of event E.

$$P(E) + P(\bar{E}) = 1$$

$$P(\bar{E}) = 1 - P(E)$$

$$P(E) = 1 - P(\bar{E})$$

Rule for complementary events: If the probability of an event or the probability of its complement is known, then the other can be found by subtracting the probability from 1.

Empirical probability: relies on actual experience to determine the likelihood of outcomes.
Given a frequency distribution, the probability of an event being in a given class is

$$P(E) = f / n$$

Where f is the frequency of the class and n is the total frequencies in the distribution.

Subjective probability: uses a probability value based on an educated guess or estimate.

4-3 The addition rules for probability

Mutually exclusive events: Two events are mutually exclusive if they cannot occur at the same time.

Addition rule1:

When two events A and B *are mutually exclusive*, the probability that A or B will occur is

$$P(A \text{ or } B) = P(A) + P(B)$$

Example2: A day of the week is selected at random. Find the probability that it is a weekend day.

Addition rule 2:
If A and B are *not mutually exclusive*, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

4-4 The multiplication rules and conditional probability
4-5

Independent events: Two events A and B are independent events if the fact that A occurs does not affect the probability of B occurring.

e.g. Drawing a card from a deck and getting a queen, replacing it and drawing a second card and getting a queen.

Multiplication rule1:
When two events are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

Dependent events: When the occurrence of the first event affects the occurrence of the second event in such a way that the probability is changed, the events are said to be *dependent events.*
e.g. Parking in a no-parking zone and getting a parking ticket.
Multiplication rule2:
When two events are dependent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \bullet P(B|A)$$

Example2.
You have a collection of 20 CDs, of which 5 are rock music. If 2 CDs are selected at random, find the probability that both are rock music.

Conditional probability:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Probability for "At least":
Use multiplication rule with the complementary event rule.

4-5 Counting rules:
In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total number of possibilities of the sequence will be

$$k_1 \cdot k_2 \cdot k_3 \cdots k_n$$

**and* means to multiply

Factorial Formula: For any counting n

$$n! = n(n-1)(n-2) \cdots 1$$

$$0! = 1$$

Combination Rule:

The number of combination of r objects selected from n objects is denoted by ${}_{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$