Chapter 8
Hypothesis Testing

Review

In Chapters 2 and 3 we used descriptive statistics when we summarized data using tools such as graphs, and statistics such as the mean and standard deviation.

Methods of inferential statistics use sample data to make an inference or conclusion about a population.

The two main activities of inferential statistics are using sample data to (1) estimate a population parameter (such as estimating a population parameter with a confidence interval), and (2) test a hypothesis or claim about a population parameter.

In Chapter 7 we presented methods for estimating a population parameter with a confidence interval, and in this chapter we present the method of hypothesis testing.

Main Objective

The main objective of this chapter is to develop the ability to conduct hypothesis tests for claims made about a population proportion p, a population mean μ, or a population standard deviation σ.

Examples of Hypotheses that can be Tested

- Genetics: The Genetics & IVF Institute claims that its XSORT method allows couples to increase the probability of having a baby girl.
- Business: A newspaper cites a PriceGrabber.com survey of 1631 subjects and claims that a majority have heard of Kindle as an e-book reader.
- Health: It is often claimed that the mean body temperature is 98.6 degrees. We can test this claim using a sample of 106 body temperatures with a mean of 98.2 degrees.

Caution

When conducting hypothesis tests as described in this chapter and the following chapters, instead of jumping directly to procedures and calculations, be sure to consider the context of the data, the source of the data, and the sampling method used to obtain the sample data.

Chapter 8
Hypothesis Testing

Key Concept

This section presents individual components of a hypothesis test. We should know and understand the following:

- How to identify the null hypothesis and alternative hypothesis from a given claim, and how to express both in symbolic form
- How to calculate the value of the test statistic, given a claim and sample data
- How to choose the sampling distribution that is relevant
- How to identify the *P*-value or identify the critical value(s)
- How to state the conclusion about a claim in simple and nontechnical terms

Definitions

A hypothesis is a claim or statement about a property of a population.

A hypothesis test is a procedure for testing a claim about a property of a population.

Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is probably not correct.

Null Hypothesis

- The null hypothesis (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is equal to some claimed value.
- We test the null hypothesis directly in the sense that we assume it is true and reach a conclusion to either reject H_0 or fail to reject H_0.

Alternative Hypothesis

- The alternative hypothesis (denoted by H_1 or H_A) is the statement that the parameter has a value that somehow differs from the null hypothesis.
- The symbolic form of the alternative hypothesis must use one of these symbols: $<$, $>$, $\neq$.

Note about Forming Your Own Claims (Hypotheses)

If you are conducting a study and want to use a hypothesis test to support your claim, the claim must be worded so that it becomes the alternative hypothesis.

Steps 1, 2, 3
Identifying H_0 and H_1

1. Identify the specific claim or hypothesis to be tested, and put in symbolic form.
2. Give the symbolic form that must be true when the original claim is false.
3. Of the two symbolic expressions obtained so far, let the alternative hypothesis H_1 be the one not containing equality, so that H_1 uses the symbol > or < or ≠. Let the null hypothesis H_0 be the symbolic expression that the parameter equals the fixed value being considered.

Example

Assume that 100 babies are born to 100 couples treated with the XSORT method of gender selection that is claimed to make girls more likely.

We observe 58 girls in 100 babies. Write the hypotheses to test the claim the "with the XSORT method, the proportion of girls is greater than the 50% that occurs without any treatment".

$$H_0 : p = 0.5$$
$$H_1 : p > 0.5$$

Step 4
Select the Significance Level α

4. Select the significance level α based on the seriousness of a type 1 error. Make α small if the consequences of rejecting a true H_0 are severe. The values of 0.05 and 0.01 are very common.

Significance Level

The significance level (denoted by α) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true (making the mistake of rejecting the null hypothesis when it is true).

This is the same α introduced in Section 7-2.

Common choices for α are 0.05, 0.01, and 0.10.

Step 5
Identify the Test Statistic and Determine its Sampling Distribution

Parameter	Sampling Distribution	Requirements	Test Statistic
Proportion p	Normal (z)	$np \geq 5$ and $nq \geq 5$	$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$
Mean μ	t	σ not known and normally distributed population *or* σ not known and $n > 30$	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
Mean μ	Normal (z)	σ known and normally distributed population *or* σ known and $n > 30$	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
St. dev. σ or variance σ^2	χ^2	Strict requirement: normally distributed population	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

Test Statistic

The test statistic is a value used in making a decision about the null hypothesis, and is found by converting the sample statistic to a score with the assumption that the null hypothesis is true.

Step 6
Find the Value of the Test Statistic, Then Find Either the *P*-Value or the Critical Value(s)

First transform the relevant sample statistic to a standardized score called the test statistic.

Then find the *P*-Value or the critical value(s).

Example

Let's again consider the claim that the XSORT method of gender selection increases the likelihood of having a baby girl.

Preliminary results from a test of the XSORT method of gender selection involved 100 couples who gave birth to 58 girls and 42 boys.

Use the given claim and the preliminary results to calculate the value of the test statistic.

Use the format of the test statistic given above, so that a normal distribution is used to approximate a binomial distribution.

Example - Continued

The claim that the XSORT method of gender selection increases the likelihood of having a baby girl results in the following null and alternative hypotheses:

$$H_0 : p = 0.5$$
$$H_1 : p > 0.5$$

We work under the assumption that the null hypothesis is true with $p = 0.5$.

The sample proportion of 58 girls in 10 births results in:

$$\hat{p} = \frac{58}{100} = 0.58$$

Example – Convert to the Test Statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.58 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{100}}} = 1.60$$

We know from previous chapters that a *z* score of 1.60 is not "unusual".

At first glance, 58 girls in 100 births does not seem to support the claim that the XSORT method increases the likelihood a having a girl (more than a 50% chance).

Types of Hypothesis Tests: Two-tailed, Left-tailed, Right-tailed

The tails in a distribution are the extreme regions bounded by critical values.

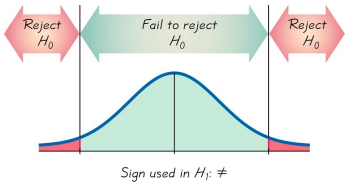
Determinations of *P*-values and critical values are affected by whether a critical region is in two tails, the left tail, or the right tail. It, therefore, becomes important to correctly ,characterize a hypothesis test as two-tailed, left-tailed, or right-tailed.

Two-tailed Test

$H_0 :=$

$H_1 :\neq$

α is divided equally between the two tails of the critical region

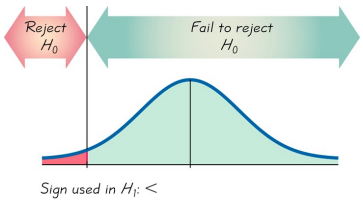


Left-tailed Test

$H_0 :=$

$H_1 :<$

All α in the left tail

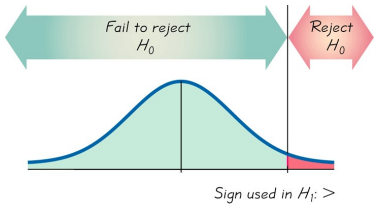


Right-tailed Test

$H_0 :=$

$H_1 :>$

All α in the right tail



P-Value

The *P*-value (or probability value) is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assuming that the null hypothesis is true.

Critical region in the left tail:	*P*-value = area to the left of the test statistic
Critical region in the right tail:	*P*-value = area to the right of the test statistic
Critical region in two tails:	*P*-value = twice the area in the tail beyond the test statistic

P-Value

The null hypothesis is rejected if the *P*-value is very small, such as 0.05 or less.

Example

The claim that the XSORT method of gender selection increases the likelihood of having a baby girl results in the following null and alternative hypotheses:

$$H_0 : p = 0.5$$
$$H_1 : p > 0.5$$

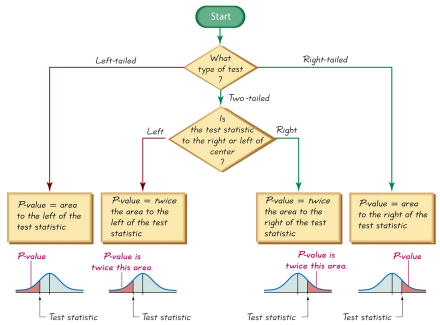
The test statistic was :

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.58 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{100}}} = 1.60$$

Example

The test statistic of $z = 1.60$ has an area of 0.0548 to its right, so a right-tailed test with test statistic $z = 1.60$ has a *P*-value of 0.0548

Procedure for Finding *P*-Values



Critical Region

The critical region (or rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis. For example, see the red-shaded region in the previous figures.

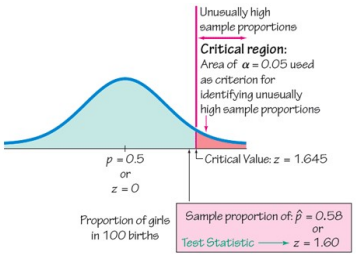
Critical Value

A critical value is any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to rejection of the null hypothesis.

The critical values depend on the nature of the null hypothesis, the sampling distribution that applies, and the significance level α.

Example

For the XSORT birth hypothesis test, the critical value and critical region for an α = 0.05 test are shown below:



Caution

Don't confuse a *P*-value with a proportion *p*.
Know this distinction:

P-value = probability of getting a test statistic at least as extreme as the one representing sample data

p = population proportion

Step 7 : Make a Decision: Reject H_0 or Fail to Reject H_0

The methodologies depend on if you are using the

P-Value method or the critical value method.

Decision Criterion

P-value Method:

Using the significance level α:

If *P*-value $\leq \alpha$, reject H_0.

If *P*-value $> \alpha$, fail to reject H_0.

Decision Criterion

Critical Value Method:

If the test statistic falls within the critical region, reject H_0.

If the test statistic does not fall within the critical region, fail to reject H_0.

Example

For the XSORT baby gender test, the test had a test statistic of $z = 1.60$ and a *P*-Value of 0.0548. We tested:

$$H_0 : p = 0.5$$
$$H_1 : p > 0.5$$

Using the *P*-Value method, we would fail to reject the null at the $\alpha = 0.05$ level.

Using the critical value method, we would fail to reject the null because the test statistic of $z = 1.60$ does not fall in the rejection region.

(You will come to the same decision using either method.)

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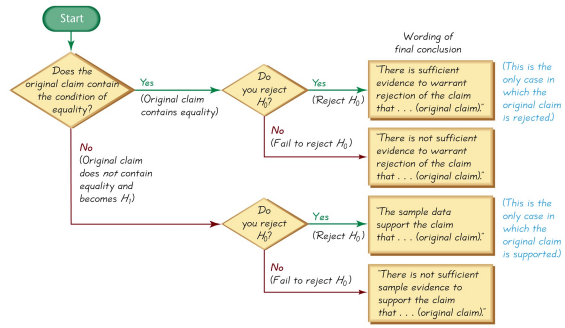
Step 8 : Restate the Decision Using Simple and Nontechnical Terms

State a final conclusion that addresses the original claim with wording that can be understood by those without knowledge of statistical procedures.

Example

For the XSORT baby gender test, there was not sufficient evidence to support the claim that the XSORT method is effective in increasing the probability that a baby girl will be born.

Wording of Final Conclusion



Caution

Never conclude a hypothesis test with a statement of "reject the null hypothesis" or "fail to reject the null hypothesis."

Always make sense of the conclusion with a statement that uses simple nontechnical wording that addresses the original claim.

Accept Versus Fail to Reject

- Some texts use "accept the null hypothesis."
- We are not proving the null hypothesis.
- Fail to reject says more correctly that the available evidence is not strong enough to warrant rejection of the null hypothesis.

Type I Error

- A Type I error is the mistake of rejecting the null hypothesis when it is actually true.
- The symbol α is used to represent the probability of a type I error.

Type II Error

- A Type II error is the mistake of failing to reject the null hypothesis when it is actually false.
- The symbol β (beta) is used to represent the probability of a type II error.

Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	**Type I error** (rejecting a true null hypothesis) P(type I error) = α	Correct decision
	We fail to reject the null hypothesis	Correct decision	**Type II error** (failing to reject a false null hypothesis) P(type II error) = β

Example

Assume that we are conducting a hypothesis test of the claim that a method of gender selection increases the likelihood of a baby girl, so that the probability of a baby girls is $p > 0.5$.

Here are the null and alternative hypotheses:

$$H_0 : p = 0.5$$
$$H_1 : p > 0.5$$

a) Identify a type I error.
b) Identify a type II error.

Example - Continued

a) A type I error is the mistake of rejecting a true null hypothesis:

We conclude the probability of having a girl is greater than 50%, when in reality, it is not. Our data misled us.

b) A type II error is the mistake of failing to reject the null hypothesis when it is false:

There is no evidence to conclude the probability of having a girl is greater than 50% (our data misled us), but in reality, the probability **is** greater than 50%.

Controlling Type I and Type II Errors

- For any fixed α, an increase in the sample size *n* will cause a decrease in *β*.
- For any fixed sample size *n*, a decrease in α will cause an increase in *β*. Conversely, an increase in *α* will cause a decrease in *β*.
- To decrease both α and *β*, increase the sample size.

Chapter 8
Hypothesis Testing

Key Concept

This section presents complete procedures for testing a hypothesis (or claim) made about a population proportion.

This section uses the components introduced in the previous section for the *P*-value method, the traditional method or the use of confidence intervals.

Key Concept

Two common methods for testing a claim about a population proportion are (1) to use a normal distribution as an approximation to the binomial distribution, and (2) to use an exact method based on the binomial probability distribution.

Part 1 of this section uses the approximate method with the normal distribution, and Part 2 of this section briefly describes the exact method.

Part 1:

Basic Methods of Testing Claims about a Population Proportion *p*

Notation

n = sample size or number of trials

$\hat{p} = \dfrac{x}{n}$

p = population proportion

$q = 1 - p$

Requirements for Testing Claims About a Population Proportion *p*

1) The sample observations are a simple random sample.

2) The conditions for a binomial distribution are satisfied.

3) The conditions $np \geq 5$ and $nq \geq 5$ are both satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$.

   Note: p is the assumed proportion not the sample proportion.

Test Statistic for Testing a Claim About a Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$$

P-values: Use the standard normal distribution (Table A-2) and refer to Figure 8-1.

Critical Values: Use the standard normal distribution (Table A-2).

Caution

Don't confuse a *P*-value with a proportion *p*.

P-value = probability of getting a test statistic at least as extreme as the one representing sample data

p = population proportion

P-Value Method

Computer programs and calculators usually provide a *P*-value, so the *P*-value method is used.

If technology is not available, see Figure 8-1 in the text.

Critical Value Method

Use the same method as described in Figure 8-2 in Section 8-2.

Example

Based on information from the National Cyber Security Alliance, 93% of computer owners believe they have antivirus programs installed on their computers.

In a random sample of 400 scanned computers, it is found that 380 of them (or 95%) actually have antivirus software programs.

Use the sample data from the scanned computers to test the claim that 93% of computers have antivirus software.

Example - Continued

Requirement check:

1. The 400 computers are randomly selected.
2. There is a fixed number of independent trials with two categories (computer has an antivirus program or does not).
3. The requirements $np \geq 5$ and $nq \geq 5$ are both satisfied with $n = 400$

$$np = (400)(0.93) = 372$$
$$nq = (400)(0.07) = 28$$

Example - Continued

P-Value Method:

1. The original claim that 93% of computers have antivirus software can be expressed as $p = 0.93$.
2. The opposite of the original claim is $p \neq 0.93$.
3. The hypotheses are written as:

$$H_0 : p = 0.93$$
$$H_1 : p \neq 0.93$$

Example - Continued

P-Value Method:

4. For the significance level, we select $\alpha = 0.05$.
5. Because we are testing a claim about a population proportion, the sample statistic relevant to this test is:

$\hat{p}$, approximated by a normal distribution

Example - Continued

P-Value Method:

6. The test statistic is calculated as:

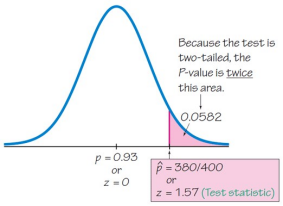
$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{380}{400} - 0.93}{\sqrt{\frac{(0.93)(0.07)}{400}}} = 1.57$$

Example - Continued

P-Value Method:

6. Because the hypothesis test is two-tailed with a test statistic of $z = 1.57$, the *P*-value is twice the area to the right of $z = 1.57$.

   The *P*-value is twice 0.0582, or 0.1164.



Example - Continued

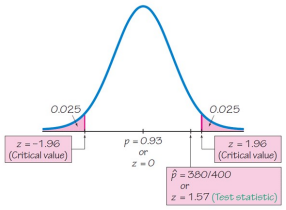
P-Value Method:

7. Because the P-value of 0.1164 is greater than the significance level of α = 0.05, we fail to reject the null hypothesis.
8. We fail to reject the claim that 93% computers have antivirus software. We conclude that there is not sufficient sample evidence to warrant rejection of the claim that 93% of computers have antivirus programs.

Example - Continued

Critical Value Method: Steps 1 – 5 are the same as for the *P*-value method.

6. The test statistic is computed to be $z = 1.57$. We now find the critical values, with the critical region having an area of $\alpha = 0.05$, split equally in both tails.



Example - Continued

Critical Value Method:

7. Because the test statistic does not fall in the critical region, we fail to reject the null hypothesis.
8. We fail to reject the claim that 93% computers have antivirus software. We conclude that there is not sufficient sample evidence to warrant rejection of the claim that 93% of computers have antivirus programs.

Chapter 8
Hypothesis Testing

Key Concept

This section presents methods for testing a claim about a population mean.

Part 1 deals with the very realistic and commonly used case in which the population standard deviation σ is not known.

Part 2 discusses the procedure when σ is known, which is very rare.

Part 1

When σ is not known, we use a "*t* test" that incorporates the Student *t* distribution.

Notation

n = sample size

$\overline{x}$ = sample mean

$\mu_{\overline{x}}$ = population mean

Requirements

1) The sample is a simple random sample.
2) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Test Statistic

$$t = \frac{\overline{x} - \mu_{\overline{x}}}{\frac{s}{\sqrt{n}}}$$

Running the Test

P-values: Use technology or use the Student *t* distribution in Table A-3 with degrees of freedom $df = n - 1$.

Critical values: Use the Student *t* distribution with degrees of freedom $df = n - 1$.

Important Properties of the Student *t* Distribution

1. The Student *t* distribution is different for different sample sizes (see Figure 7-5 in Section 7-3).
2. The Student *t* distribution has the same general bell shape as the normal distribution; its wider shape reflects the greater variability that is expected when *s* is used to estimate σ.
3. The Student *t* distribution has a mean of $t = 0$.
4. The standard deviation of the Student *t* distribution varies with the sample size and is greater than 1.
5. As the sample size *n* gets larger, the Student *t* distribution gets closer to the standard normal distribution.

Example

Listed below are the measured radiation emissions (in W/kg) corresponding to a sample of cell phones.

Use a 0.05 level of significance to test the claim that cell phones have a mean radiation level that is less than 1.00 W/kg.

0.38	0.55	1.54	1.55	0.50	0.60	0.92	0.96	1.00	0.86	1.46

The summary statistics are: $\overline{x} = 0.938$ and $s = 0.423$.

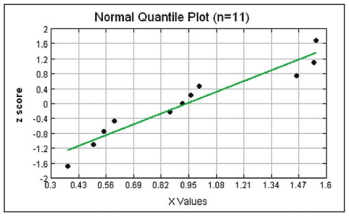
Example - Continued

Requirement Check:

1. We assume the sample is a simple random sample.
2. The sample size is n = 11, which is not greater than 30, so we must check a normal quantile plot for normality.

Example - Continued

The points are reasonably close to a straight line and there is no other pattern, so we conclude the data appear to be from a normally distributed population.



Example - Continued

Step 1: The claim that cell phones have a mean radiation level less than 1.00 W/kg is expressed as μ < 1.00 W/kg.

Step 2: The alternative to the original claim is $\mu \geq$ 1.00 W/kg.

Step 3: The hypotheses are written as:

$$H_0 : \mu = 1.00 \text{ W/kg}$$
$$H_1 : \mu < 1.00 \text{ W/kg}$$

Example - Continued

Step 4: The stated level of significance is α = 0.05.

Step 5: Because the claim is about a population mean μ, the statistic most relevant to this test is the sample mean: $\bar{x}$.

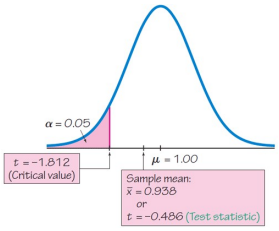
Example - Continued

Step 6: Calculate the test statistic and then find the P-value or the critical value from Table A-3:

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}} = \frac{0.938 - 1.00}{\frac{0.423}{\sqrt{11}}} = -0.486$$

Example - Continued

Step 7: Critical Value Method: Because the test statistic of t = –0.486 does not fall in the critical region bounded by the critical value of t = –1.812, fail to reject the null hypothesis.



Example - Continued

Step 7: *P*-value method: Technology, such as a TI-83/84 Plus calculator can output the *P*-value of 0.3191. Since the *P*-value exceeds $\alpha = 0.05$, we fail to reject the null hypothesis.

TI-83/84 PLUS

```
T-Test
 μ<1
 t=-.4848517201
 p=.3191133677
 x̄=.9381818182
 Sx=.4228668391
 n=11
```


Example

Step 8: Because we fail to reject the null hypothesis, we conclude that there is not sufficient evidence to support the claim that cell phones have a mean radiation level that is less than 1.00 W/kg.

Finding *P*-Values

Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the *P*-value corresponding to the given results.

a) In a left-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -2.007$.
b) In a right-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = 1.222$.
c) In a two-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -3.456$.

Part 2

When σ is known, we use test that involves the standard normal distribution.

In reality, it is very rare to test a claim about an unknown population mean while the population standard deviation is somehow known.

The procedure is essentially the same as a *t* test, with the following exception:

Test Statistic for Testing a Claim About a Mean (with σ Known)

The test statistic is: $$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}}$$

The *P*-value can be provided by technology or the standard normal distribution (Table A-2).

The critical values can be found using the standard normal distribution (Table A-2).

Example

If we repeat the cell phone radiation example, with the assumption that $\sigma = 0.480$ W/kg, the test statistic is:

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{0.938 - 1.00}{\frac{0.480}{\sqrt{11}}} = -0.43$$

The example refers to a left-tailed test, so the *P*-value is the area to the left of $z = -0.43$, which is 0.3336 (found in Table A-2).

Since the *P*-value is large, we fail to reject the null and reach the same conclusion as before.

Chapter 8
Hypothesis Testing

8-1 Review and Preview

8-2 Basics of Hypothesis Testing

8-3 Testing a Claim about a Proportion

8-4 Testing a Claim About a Mean

8-5 Testing a Claim About a Standard Deviation or Variance

Key Concept

This section introduces methods for testing a claim made about a population standard deviation σ or population variance σ^2.

The methods of this section use the chi-square distribution that was first introduced in Section 7-4.

Requirements for Testing Claims About σ or σ^2

n = sample size

s = sample standard deviation

s^2 = sample variance

σ = claimed value of the population standard deviation

σ^2 = claimed value of the population variance

Requirements

1. The sample is a simple random sample.
2. The population has a normal distribution.

   (This is a much stricter requirement than the requirement of a normal distribution when testing claims about means.)

Chi-Square Distribution

Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

P-Values and Critical Values for Chi-Square Distribution

- *P*-values: Use technology or Table A-4.
- Critical Values: Use Table A-4.
- In either case, the degrees of freedom = n –1.

Caution

The χ^2 test of this section is not robust against a departure from normality, meaning that the test does not work well if the population has a distribution that is far from normal.

The condition of a normally distributed population is therefore a much stricter requirement in this section than it was in Section 8-4.

Properties of Chi-Square Distribution

- All values of χ^2 are nonnegative, and the distribution is not symmetric (see the Figure on the next slide).
- There is a different distribution for each number of degrees of freedom.
- The critical values are found in Table A-4 using $n - 1$ degrees of freedom.

Properties of Chi-Square Distribution

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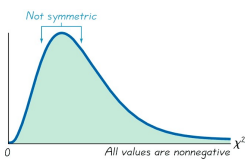
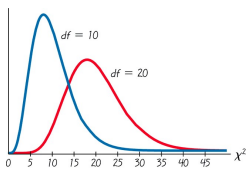


Chi-Square Distribution for 10 and 20 df



Different distribution for each number of df.

Example

Listed below are the heights (inches) for a simple random sample of ten supermodels.

Consider the claim that supermodels have heights that have much less variation than the heights of women in the general population.

We will use a 0.01 significance level to test the claim that supermodels have heights with a standard deviation that is less than 2.6 inches.

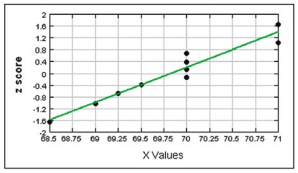
70	71	69.25	68.5	69	70	71	70	70	69.5

Summary Statistics: $s^2 = 0.7997395$ and $s = 0.8942816$

Example - Continued

Requirement Check:

1. The sample is a simple random sample.
2. We check for normality, which seems reasonable based on the normal quantile plot.



Example - Continued

Step 1: The claim that "the standard deviation is less than 2.6 inches" is expressed as $\sigma < 2.6$ inches.

Step 2: If the original claim is false, then $\sigma \geq 2.6$ inches.

Step 3: The hypotheses are:

$$H_0 : \sigma = 2.6 \text{ inches}$$
$$H_1 : \sigma < 2.6 \text{ inches}$$

Example - Continued

Step 4: The significance level is $\alpha = 0.01$.

Step 5: Because the claim is made about σ, we use the chi-square distribution.

Example - Continued

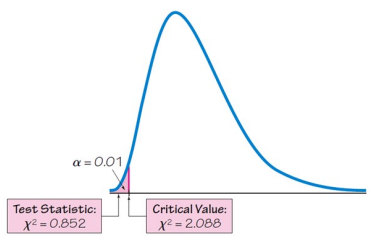
Step 6: The test statistic is calculated as follows:

$$x^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(0.7997395)^2}{2.6^2} = 0.852$$

with 9 degrees of freedom.

Example - Continued

Step 6: The critical value of $\chi^2 = 2.088$ is found from Table A-4, and it corresponds to 9 degrees of freedom and an "area to the right" of 0.99.



Example - Continued

Step 7: Because the test statistic is in the critical region, we reject the null hypothesis.

There is sufficient evidence to support the claim that supermodels have heights with a standard deviation that is less than 2.6 inches.

Heights of supermodels have much less variation than heights of women in the general population.

Example - Continued

P-Value Method:

P-values are generally found using technology, but Table A-4 can be used if technology is not available.

Using a TI-83/84 Plus, the *P*-value is 0.0002897.

TI-83/84 PLUS

```
S² TEST
SIGMA²<6.76
X²=.851516185
P=2.897435436E-4
```


Example - Continued

P-Value Method:

Since the *P*-value = 0.0002897, we can reject the null hypothesis (it is under the 0.01 significance level).

We reach the same exact conclusion as before regarding the variation in the heights of supermodels as compared to the heights of women from the general population.