Name Key

MATH 119 EXAM 2
Spring 2018
Form A

<u>Show all work on this exam form.</u> Free response questions REQUIRE that you show supporting work to get full credit.

Please round your answers to <u>four digits</u> after the decimal when possible. Make sure to BOX your final answers.

All questions are worth 5 points unless noted otherwise.

A

Use the following information for questions 1-3 (14 total points)

A professor has noticed that even though attendance is not a component for grade in his class, the students who attend regularly obtain better grades. In fact, 40% of the students who attend regularly receive As in the course, while only 10% of the students who do not attend regularly receive As in the course. About 70% of the students attend class regularly. Let event R be attending regularly and event A be receiving an A in the course.

$P(A|R) = .4 \qquad P(A|R^c) = .1 \qquad P(R) = .7$

1. Complete a tree diagram using the **correct probability notation** and values:

attend reg
P(R) $\underline{.7}$

- receive A — P($A|R$) $\underline{.4}$ → P(A and R) $\underline{.28}$ $(.7)(.4)$
- no A — P($A^c|R$) $\underline{.6}$ → P(A^c and R) $\underline{.42}$ $(.7)(.6)$

not attend reg
P(R^c) $\underline{.3}$

- A — P($A|R^c$) $\underline{.1}$ → P(A and R^c) $\underline{.03}$ $(.3)(.1)$
- no A — P($A^c|R^c$) $\underline{.9}$ → P(A^c and R^c) $\underline{.27}$ $(.3)(.9)$

2. What is the probability a student in this professor's course receives an A?

	A	A^c	tot
R	.28	.42	.7
R^c	.03	.27	.3
tot	.31	.69	1

$P(A) = \boxed{.31}$

3. Given that a student received an A, what is the probability they attended class regularly?

$$P(R|A) = \frac{P(R \text{ and } A)}{P(A)} = \frac{.28}{.31} = \boxed{.9032}$$

BINOM

4. Suppose that 15% of items created in a particular factory have cosmetic defects. If you randomly selected 20 items for inspection, what is the probability that at least 5 have cosmetic defects?

$p = .15 \qquad n = 20$

$\underbrace{0\ 1\ 2\ 3\ 4}\ \boxed{5 \ldots\ 20}$

A. 0.9327 B. 0.8298 C. 0.9781 D. 0.0673 E. 0.1702

1-binomcdf(20,.15,4)
$= 1 - .8298$
$= .1702$

NORMAL

A

Use the following information to answer questions 5 through 8

Suppose the yearly rainfall totals for a city in northern California follow a normal distribution with a mean of 18 inches and a standard deviation of 6 inches.

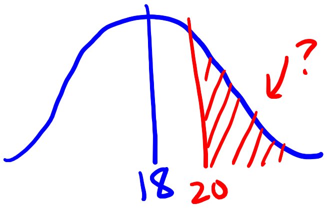
$\mu = 18 \quad \sigma = 6$

5. What is the probability that the total rainfall will be less than 10 inches?

normalcdf(-E99, 10, 18, 6)
Lower upper μ σ

$= \boxed{.0912}$

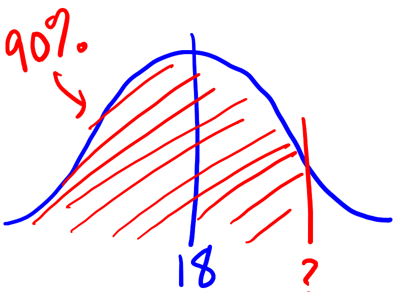
6. What is the probability that the total rainfall will be at least 20 inches?



normalcdf(20, E99, 18, 6)
Lower upper μ σ

$= \boxed{.3694}$

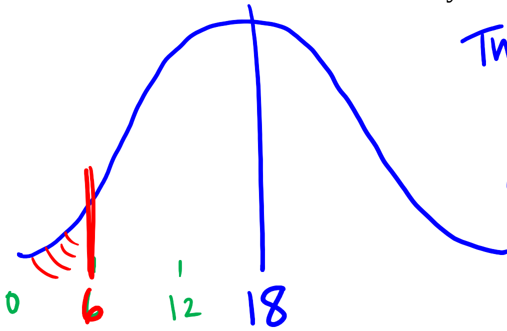
7. Suppose a particularly rainy year was in the 90th percentile for total rainfall. How many inches of rain fell that year?



invNorm(.9, 18, 6)
Left area μ σ

$= \boxed{25.6893 \text{ inches}}$

8. Suppose that last year, the rainfall was only 6 inches. You work for a local newspaper and your editor has asked you to write a story about how terrible the drought is and how abnormal the situation is. Write a few sentences you could use to explain the statistical facts to your readers (and your editor). Be sure to comment on whether or not you agree that the situation is terribly abnormal.



Two options: ① $Z = \frac{6-18}{6} = -2$

or ② $P(x < 6)$

normalcdf(-E99, 6, 18, 6) = 0.0228

a z-score of -2 is pretty unusual. also there is only a 2.28% chance of a drought that bad or worse, which is very unlikely.

A

BINOM or sampling w/ replacement

9. Shaun has been studying all night for an exam and is afraid he will oversleep for his early morning stats class and miss the exam. He sends text messages to three of his friends asking them to call him before his class to make sure he is awake. Suppose that the probability that each friend will call is 0.7 and is independent for the three friends. What is the probability that none of his friends call and he misses the exam?

$p = .7$ $n = 3$

$P(X=0) = \text{binompdf}(3, .7, 0) = \boxed{0.027}$

OR

$\underbrace{(.3)}_{\text{No}} \underbrace{(.3)}_{\text{No}} \underbrace{(.3)}_{\text{No}} = \boxed{.027}$

10. Suppose that at a certain college, 7% of full-time students take 3 courses this semester, 14% take 4 courses, 52% take 5 courses, 25% take 6 courses, and 2% take 7 courses.

X	P(X)
3	.07
4	.14
5	.52
6	.25
7	.02

a. Find the probability that a full-time student at this college is taking less than 5 courses this semester.

$.07 + .14 = \boxed{.21}$

b. Find the expected number of courses a full-time student at this college will be enrolled in this semester.

$E(X) = \sum x \cdot p(x) = 3(.07) + 4(.14) + 5(.52) \ldots = \boxed{5.01}$

OR
L1 = X
L2 = P(X)
1-var-stats L1, L2 $\bar{x} = \boxed{5.01 \text{ classes}}$

11. Explain which of the conditions for a binomial experiment is NOT met for each of the following random variables:

a. A football team plays 12 games in its regular season. X = the number of games won.

independence/success doesn't stay same, some teams may be easier and they may have better odds at home v. away.

b. A woman buys a lottery ticket every week for which the probability of winning anything at all is 1/10. She continues to buy them until she has won three times. X = the number of tickets she buys.

not a fixed # of trials

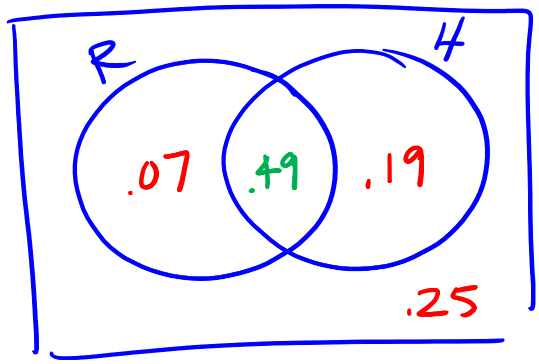
P(R) P(H)

Use the following information for the questions 14 through 19 (23 total points)
Fifty-six percent of all American workers have a workplace retirement plan (event R), 68% have health insurance, and 49% have both benefits.

P(R and H)

12. Create and fill in a contingency table or Venn diagram to display this situation.

	R	R^c	total
H	.49	.19	.68
H^c	.07	.25	.32
total	.56	.44	1



13. What is the probability that a randomly selected worker has neither employer-sponsored health insurance nor a retirement plan?

H^c R^c

$P(H^c \text{ and } R^c)$

$= \boxed{.25}$

14. What is the probability a worker has either health insurance or a retirement plan?

H R

$P(H \text{ or } R) = P(H) + P(R) - P(H \text{ and } R)$

$= .68 + .56 - .49 = \boxed{.75}$

15. What is the probability that a worker has health insurance given they have a retirement plan?

H R

$P(H|R) = \dfrac{P(H \text{ and } R)}{P(R)} = \dfrac{.49}{.56} = \boxed{.875}$

16. Are having health insurance and a retirement plan independent? Explain using probabilities.

① $P(H \text{ and } R) \neq P(H) \cdot P(R)$

$.49 \neq (.68)(.56)$

$.49 \neq .3808$

② $P(H|R) \neq P(H)$

$.875 \neq .68$

No because of ① or ②. ☺

If someone has retirement they are more likely to have health ins.

17. Are having health insurance and a retirement plan disjoint? Explain using probabilities.

$P(H \text{ and } R) \neq 0$

$.49 \neq 0$

No because a worker can have health ins and retirement!

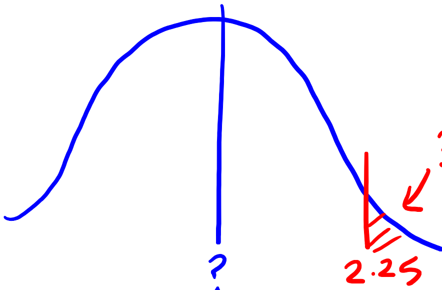
A

Use the following information for the questions 20 through 22 forgot "normal" OOPS

A machine is used to fill soda bottles in a factory. The bottles are labeled as containing 2.0 liters, but extra room at the top of the bottle allows for a maximum of 2.25 liters of soda before the bottle overflows. The standard deviation of the amount of soda put into the bottles by the machine is known to be 0.15 liter. $\sigma = 0.15$

18. If management requires that no more than 3% of bottles should be overfilled (putting more than the maximum 2.25 liters, resulting in soda being spilled on the machine), the machine should be set to fill the bottles with what mean amount? TRICKY INVERSE. Right side

$\mu = ?$



① $z = \text{invNorm}(.97, 0, 1)$ left side μ σ

$z = 1.88$

② $X = z(\sigma) + \mu$

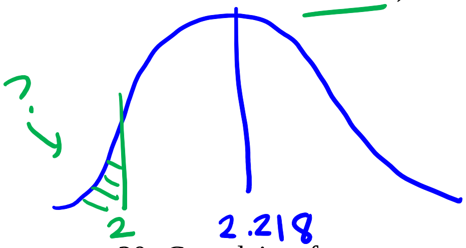
$2.25 = 1.88(.15) + \mu$

$2.25 = .282 + \mu$

$-.282 \quad -.282$

$\boxed{\mu = 2.218\text{ L}}$

19. If you used the mean from (a), what percent of bottles will be under-filled (less than the advertised 2.0 liters)?



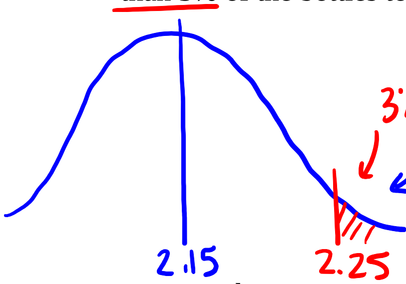
normalcdf(-E99, 2, 2.218, 0.15)

$= \boxed{.0731}$

20. Complaints from consumers about under-filled bottles leads the company to set the mean amount to 2.15 liters. In this situation, what standard deviation would allow for no more than 3% of the bottles to be overfilled? $\sigma = ?$ TRICKY INV

$\mu = 2.15$

over 2.25L



① invNorm(.97) $= 1.88$

② $2.25 = 1.88\sigma + 2.15$

$-2.15 \quad -2.15$

$\frac{0.1}{1.88} = \frac{1.88\sigma}{1.88}$

$\boxed{\sigma = .0532\text{ L}}$

21. Suppose that Mama Cat has a litter with 7 kittens. 3 of the kittens are black and the other 4 kittens are gray. If you randomly select two kittens to adopt, what is the probability that at least one of them will be black? sampling w/out replacement

BBB
GGGG

P(at least 1) =

$1 - P(\text{none}) = 1 - .2857 = \boxed{.7143}$

$P(\text{none}) = \underset{G}{\frac{4}{7}} \cdot \underset{G}{\frac{3}{6}} = .2857$

normal ~~discrete~~ binom

22. A book is randomly chosen from a library shelf. For each of the following characteristics of the book, decide whether the characteristic is a continuous or discrete random variable.

    a. Weight of the book continuous

    b. Number of chapters in the book discrete

    c. Width of the book continuous

    d. Year the book was published discrete

n=110 p=.55

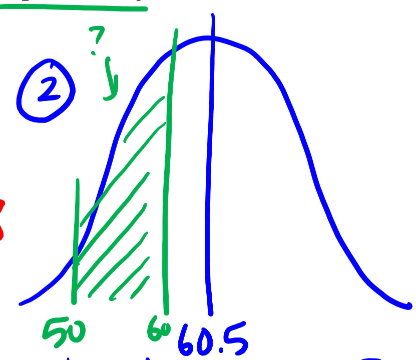
23. It is known that, in Toronto, Canada, 55% of people pass the drivers' road test. Suppose that every day, 110 people independently take the test.
Using the Normal Approximation to the Binomial, determine the probability that between 50 and 60 people pass the test. normalcdf

① $\mu = np = 110(.55) = 60.5$

$\sigma = \sqrt{npq} = \sqrt{110(.55)(.45)} = 5.2178$

$q = 1-p$

②



normalcdf(50, 60, 60.5, 5.2178)

$= \boxed{.4397}$

Statistics 119 Formulas for the Semester

Probability Formulas:

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A\ and\ B) = P(A \cap B) = P(A \mid B) \bullet P(B) \qquad P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Mean and standard deviation of a discrete random variable

$$E(X) = \mu = \sum xp(x) \qquad \sigma = \sqrt{\left(\sum (x-\mu)^2 p(x)\right)} = \sqrt{\sum x^2 p(x) - \mu^2}$$

Binomial Probability Function:

$$P(X = k) = {}_nC_k\, p^k q^{n-k}$$

$$Mean = \mu_X = np$$

$$Standard\ \ Deviation = \sigma_X = \sqrt{npq} \qquad z^* = \frac{x-np}{\sqrt{npq}} = \frac{x-\mu_X}{\sigma_X}$$

Sampling Distribution of a Sample Proportion:

$$Mean = \mu_{\hat{p}} = p$$

$$Standard\ Deviation = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{pq}{n}} \qquad Z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = Z = \frac{\hat{p}-\mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

Normal Distribution:

Direct calculation: $z = \dfrac{x-\mu}{\sigma}$ *Inverse calculation:* $x = z(\sigma) + \mu$

Mean and standard deviation of sample mean: $\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Direct calculation: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ *Inverse calculation:* $\bar{x} = z\left(\dfrac{\sigma}{\sqrt{n}}\right) + \mu$

	Hypothesis Test	Confidence Interval	Sample Size
For proportions	$z = \dfrac{\hat{p}-p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$	$\hat{p} \pm Z^* \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$	$n = \dfrac{(Z^*)^2\,\hat{p}\hat{q}}{(ME)^2}$
For means (σ known)	$Z = \dfrac{\bar{x}-\mu}{\left(\sigma/\sqrt{n}\right)}$	$\bar{x} \pm Z^* \left(\dfrac{\sigma}{\sqrt{n}}\right)$	$n = \left(\dfrac{Z^*\sigma}{ME}\right)^2$
For means (σ unknown)	$t = \dfrac{\bar{x}-\mu}{\left(S/\sqrt{n}\right)}$	$\bar{x} \pm t^* \left(\dfrac{S}{\sqrt{n}}\right)$	$n = \left(\dfrac{(t^*)(s)}{ME}\right)^2$

Table entry for z is the area under the standard normal curve to the left of z.

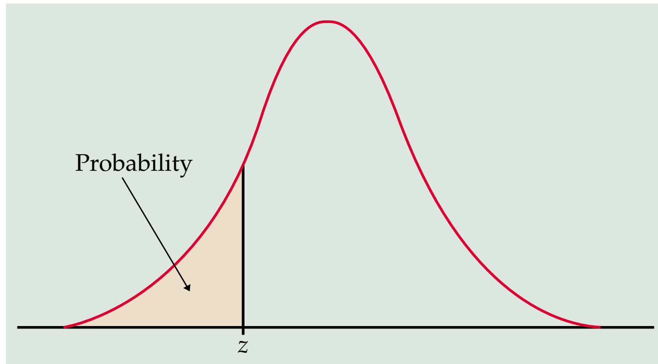


TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard normal curve to the left of z.

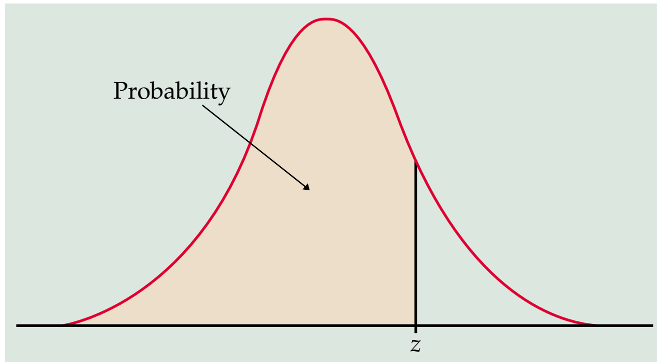


TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998