

Name Key

**STATS EXAM 3
Fall 2018
Form A**

- **Show as much work as possible for partial credit!**
- Please round your answers to four digits after the decimal when possible. Make sure to put your final answers neatly within the answer boxes (when provided).
- **Answers may vary between calculator and table, so keep that in mind on multiple choice problems.**
- If you get stuck on a multiple part question MAKE UP AN ANSWER and continue with that value. I grade for consistency between parts of a problem.
- **All questions are worth 4 points unless noted otherwise.**

A

A

1. (20 points) Dirt bikes are simpler and lighter motorcycles that are designed for off-road events. Specifications for dirt bikes can be found through Motorcycle USA. A random sample of 30 dirt bikes has a mean fuel capacity of 1.81 gallons with a standard deviation of 0.74 gallons. At the 5% significance level, is there sufficient evidence to conclude that the mean fuel tank capacity of all dirt bikes is less than 2 gallons?

$$n = 30$$

$$\bar{x} = 1.81$$

$$s = 0.74$$

$$\alpha = .05$$

$$\mu = 2$$

A. State the appropriate null and alternative hypothesis.

$$H_0: \mu = 2 \quad H_a: \mu < 2 \quad T\text{Test!}$$

B. Calculate the test statistic.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.81 - 2}{0.74/\sqrt{30}} = -1.406$$

C. Calculate the corresponding p-value.

CALC →

$$p\text{value} = .0851$$

TABLE

$$.05 < p\text{value} < .1$$

D. Make and justify a statistical decision using a significance level of 5%.

$$p\text{value} > \alpha = .05 \quad \text{FTRN!}$$

E. Interpret your decision in the context of the problem.

There is not enough evidence to conclude that the mean fuel tank capacity of all dirt bikes is less than 2 gallons.

Use the following information to answer the next two questions

Suppose it is known that 63% of American adults have an organ donor card. A random sample of 240 American adults is selected.

$$n = 240$$

$$p = .63$$

2. Fill in the blanks to describe the distribution of the proportion of adults in the sample of 240 Americans who have an organ donor card

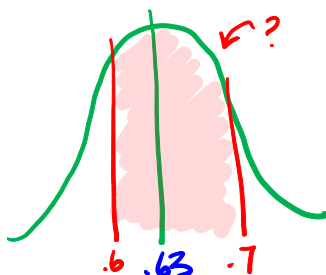
$$\hat{p} \sim \text{AN} (.63, .0312)$$

$$\mu_{\hat{p}} = p = .63$$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.63)(.37)}{240}}$$

3. (5 points) Find the probability that between 60% and 70% of the sample of 240 Americans will have an organ donor card.

$$P(.6 < \hat{p} < .7)$$



$$\text{normalcdf}(.6, .7, .63, .0312)$$

$$= .8194$$

A

Use the following information to answer the next three questions

The manager of a local fast food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.

$n=80$

$X=23$

4. (5 points) Construct a 95% confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

1-Prop Z Int

$$\hat{p} = \frac{23}{80} = .2875$$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = .2875 \pm 1.96 \sqrt{\frac{(.2875)(.7125)}{80}}$$

$$= .2875 \pm .0992 = (.1883, .3867)$$

5. (5 points) Interpret the interval you found above.

We are 95% confident that the proportion of all customers who shadily fill their water cup with a soft drink is between .1883 and .3867.

6. (2 points) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant \$0.25. Suppose that in the month of June, 3000 customers ask for a water cup when placing an order. Use the confidence interval in part (a) to give an interval estimate for the cost to the restaurant for the month of June from customers who ask for a water cup but fill the cup with a soft drink.

PROPORTION: (.1883, .3867)

#: $3000(.1883, .3867) = (565, 1161)$ shady customers

\$: $0.25(565, 1161) = \$148.75 \text{ to } \290.25

↘ 25¢ per Shady customer ↗

7. Toilets with an automatic sensor flushing system use motion as their evidence. Considering a null hypothesis that you are not done pooping, which of the following would correctly describe a Type II error in this situation?

- (A) The toilet doesn't flush when you are done pooping.
 B. The toilet doesn't flush when you are not done pooping.
 C. The toilet flushes when you are done pooping.
 D. The toilet flushes when you are not done pooping.

Type II: FTRN when H_0 false

H_0 : not done
 Decision doesn't flush
 H_A : done
 Truth

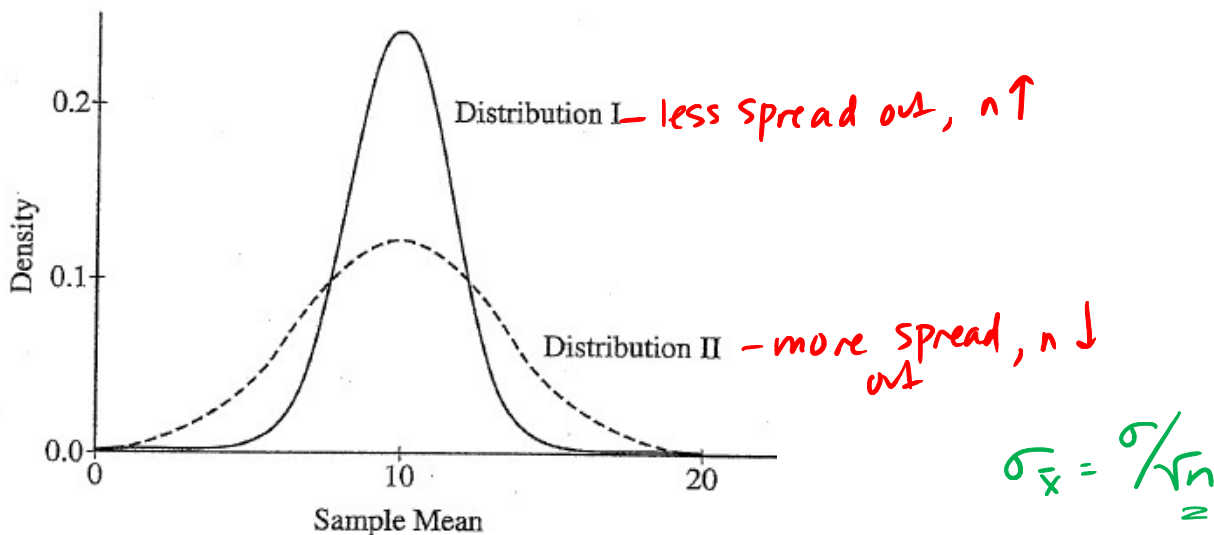
A

8. Over the past several years 58% of adults believed that global warming was real. A 2012 Gallup Poll of 1014 adults found that 52% of the sample believed that global warming was real. Is there enough evidence to show the proportion who believes in global warming has changed? $\rightarrow \neq$

Identify the appropriate null and alternative hypotheses.

- A. $H_0 : p = 0.52$ $H_a : p \neq 0.52$
 B. $H_0 : p = 0.58$ $H_a : p < 0.58$
 C. $H_0 : p = 0.58$ $H_a : p \neq 0.58$
 D. $H_0 : p = 0.52$ $H_a : p > 0.52$

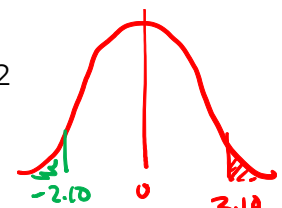
9. The graphs of the sampling distributions, I and II, of the sample mean of the same random variable for samples of two different sizes are shown below. Which of the following statements must be true about the sample sizes?



- A. The sample size of I is less than the sample size of II.
 B. The sample size of I is greater than the sample size of II.
 C. The sample size of I is equal to the sample size of II.
 D. The sample size does not affect the sampling distribution.
 E. The sample sizes cannot be compared based on these graphs.
10. Which of the following will result in a narrower confidence interval for a mean? Circle the correct answers:
 Increasing / Decreasing the sample size.
 Increasing / Decreasing the confidence level.
11. In performing a two-sided hypothesis test, you calculated that the test statistic $z = 2.10$. What is the appropriate p-value?

- A. 0.0179 B. 0.5 C. 0.9821 D. 0.0358 E. 1.9642

① look up -2.10
 ② $\times 2$ since two-sided



A

12. (5 points) A researcher wishes to estimate the population proportion of U.S. adults who are overweight. They wish to estimate the proportion to within 4.5% with 95% confidence. How many individuals should be included in the sample?

$n = ?$

not given

$$n = \frac{(Z^*)^2 \hat{p} \hat{q}}{(ME)^2} = \frac{(1.96)^2 (.5)(.5)}{(.045)^2} = 474.27$$

475 individuals

13. In a random sample of 14 iPads, the sample mean battery life was calculated to be 10.2 hours with a standard deviation of 2.5 hours. Which of the following formulas would you use to create a 95% confidence interval for the population mean battery life of an iPad?

A. $\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$

B. $\bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}} \right)$

C. $\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}} \right)$

- D. None of the above, the assumptions are not met to create a confidence interval. *n isn't large enough!*

14. A 95% confidence interval is computed to estimate the mean household income for a city. Which of the following values will definitely be within the limits of this confidence interval?

A. The population mean

B. The sample mean

C. The standard deviation of the sample mean

D. None of the above

15. Suppose two researchers want to estimate the proportion of American college students who favor abolishing the penny. They both want to have about the same margin of error to estimate this proportion. However, Researcher 1 wants to estimate with 99% confidence and Researcher 2 wants to estimate with 95% confidence. Which researcher would need more students for her study in order to obtain the desired margin of error?

A. Researcher 1.

B. Researcher 2.

C. Both researchers would need the same number of subjects.

D. It is impossible to obtain the same margin of error with the two different confidence levels.

Super hard!

A

16. As lab partners, Sally and Betty collected data for a significance test. Both calculated the same z-test statistic, but Sally found the results were significant at the $\alpha = 0.05$ level while Betty found that the results were not. When checking their results, the women found that the only difference in their work was that Sally had used a two-sided test, while Betty used a one-sided test. Which of the following could have been their test statistic?

- A. -1.980
B. -1.690
C. 1.340
D. 1.690
E. 1.780



must be ... 0.05!
Betty must have done RIGHT TAIL
one = .0239
two = .0478
one = .0455
two = .0910
one = .0901
two = .1802
one = .0375
two = .0750

17. A survey was conducted to estimate the proportion of adults who say it is acceptable to check personal e-mail while at work. A 95% confidence interval was calculated to be (0.633, 0.691). Identify the margin of error.

A. 0.029

B. 0.058

C. 0.662



$$ME = \frac{.691 - .633}{2} = .029$$

18. What critical value (table value) would you use to create a 90% confidence interval for the true average weight of a watermelon if you used a sample of size 20?

$$\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}} \right)$$

$$t^* = ?$$

$$df = 19$$

$$t^* = 1.729$$

Use the following information to answer the following two questions

Do people lie about voting? A survey was conducted to determine the proportion of eligible voters who said that they voted in the recent presidential election. Based on the response of 1002 randomly selected adults a 96% confidence interval was calculated to be (0.671, 0.728).

inside

$$H_0: p = .68$$

$$H_A: p \neq .68$$

19. Voting records show that 68% of eligible voters actually did vote. Use the above confidence interval to test the null hypothesis that people do not lie about voting (so the proportion who say they vote is the same as the proportion who actually vote)

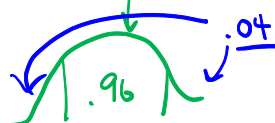
- A. There is enough evidence to conclude that the people do not lie about voting.
B. There is not enough evidence to conclude that the people do not lie about voting.
C. There is enough evidence to conclude that the people lie about voting.
D. There is not enough evidence to conclude that the people lie about voting.

FTRN

H_A

20. (2 points) What is the level of significance of the above hypothesis test?

- A. 0.02
B. 0.04
C. 0.05
D. 0.06
E. There is not enough information to determine the significance level.



A

21. A researcher conducts an experiment on human memory and recruits 15 people to participate in her study. She performs the experiment and analyzes the results. She obtains a p-value of 0.17. Which of the following is a reasonable interpretation of her results?
- huge(ish)! FTRN*
- A. This proves that her experimental treatment has no effect on memory.
 - ☒ B. There could be a treatment effect, but the sample size was too small to detect it.
 - ~~C.~~ She should reject the null hypothesis.
 - ~~D.~~ There is evidence of a small effect on memory by her experimental treatment.

True / False, each worth 2 points.

22. The ~~null~~ hypothesis should be what you are trying to prove.

alternative

TRUE / ☒ FALSE

23. If we increase the Type I error in our hypothesis test, we will also increase the power.

↑α ↑power

☒ TRUE / FALSE

24. If we increase the sample size, the sampling distribution of the sample mean will be less spread out.

☒ TRUE / FALSE

↓
 $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

Goldilocks Survey

Difficulty: I felt that this exam was.... too easy / just right / too hard.

Length: I felt that this exam was.... too short / just right / too long.

Statistics 119 Formulas for the Semester

Probability Formulas:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \text{ and } B) = P(A \cap B) = P(A|B) \bullet P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Mean and standard deviation of a discrete random variable

$$E(X) = \mu = \sum xp(x) \qquad \sigma = \sqrt{\left(\sum (x - \mu)^2 p(x) \right)} = \sqrt{\sum x^2 p(x) - \mu^2}$$

Binomial Probability Function:

$$P(X = k) = {}_n C_k p^k q^{n-k}$$

$$\text{Mean} = \mu_x = np$$

$$\text{Standard Deviation} = \sigma_x = \sqrt{npq}$$

$$z^* = \frac{x - np}{\sqrt{npq}} = \frac{x - \mu_x}{\sigma_x}$$

Sampling Distribution of a Sample Proportion:

$$\text{Mean} = \mu_{\hat{p}} = p$$

$$\text{Standard Deviation} = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{pq}{n}}$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

Normal Distribution:

$$\text{Direct calculation: } z = \frac{x - \mu}{\sigma}$$

$$\text{Inverse calculation: } x = z(\sigma) + \mu$$

$$\text{Mean and standard deviation of sample mean: } \mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\text{Direct calculation: } z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$\text{Inverse calculation: } \bar{x} = z \left(\frac{\sigma}{\sqrt{n}} \right) + \mu$$

	Hypothesis Test	Confidence Interval	Sample Size
For proportions	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$	$\hat{p} \pm Z^* \sqrt{\frac{\hat{p} \hat{q}}{n}}$	$n = \frac{(Z^*)^2 \hat{p} \hat{q}}{(ME)^2}$
For means (σ known)	$Z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}} \right)}$	$\bar{x} \pm Z^* \left(\frac{\sigma}{\sqrt{n}} \right)$	$n = \left(\frac{Z^* \sigma}{ME} \right)^2$
For means (σ unknown)	$t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}} \right)}$	$\bar{x} \pm t^* \left(\frac{S}{\sqrt{n}} \right)$	$n = \left(\frac{(t^*)(s)}{ME} \right)^2$



Table entry for z is the area under the standard normal curve to the left of z .

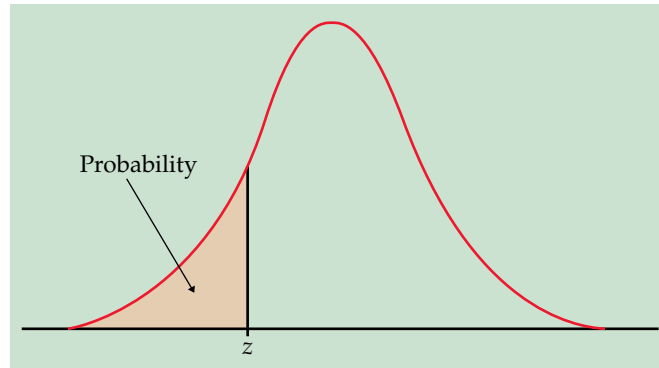


TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0006	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Probability

z

TABLE A Standard normal probabilities (*continued*)

[illegible]