STATS EXAM 3
Spring 2018
Form A

Show all work on this exam form. **Free response questions REQUIRE that you show supporting work to get full credit.**

Please round your answers to <u>four digits</u> after the decimal when possible. Make sure to BOX your final answers.

All questions are worth 5 points unless noted otherwise.

A

A

1. (15 points) A person read that the average number of hours an adult sleeps on Friday night to Saturday morning was 7.2 hours. The researcher feels that college students do not sleep 7.2 hours on average. The researcher randomly selected 15 students and found that they slept an average of 7.9 hours. The standard deviation of the sample is 1.2 hours. At a significance level of 0.01, is there enough evidence to say that college students do not sleep 7.2 hours on average?

   a. State the appropriate null and alternative hypothesis.

   b. Calculate the test statistic. Be sure to specific if it is a z or t.

   c. Calculate the corresponding p-value (or range for p-values).

   d. Make and justify a statistical decision using a significance level of 1%.

   e. Interpret your decision in the context of the problem.

2. Which of the following does **not** affect the width of a confidence interval for the population mean?
   A. The variability of the data.
   B. The sample size.
   C. The confidence level of the interval.
   D. The sample mean.

3. (10 points) A certain medical procedure produces side effects in 25% of the patients who receive it. A random sample of 130 adults who receive this procedure is selected.

a. Describe the distribution of the **proportion** of patients in the sample who have side effects from the medical procedure.

A. $X \sim AN(32.5, 24.375)$ B. $\hat{p} \sim AN(32.5, 4.9371)$
C. $\hat{p} \sim AN(0.25, 0.0014)$ D. $\hat{p} \sim AN(0.25, 0.0380)$

b. What is the probability that **at least 34%** of the sample of 130 patients have side effects from the procedure? Calculate this using the distribution selected above.

Use the following information to answer question 4 – 7 (13 points total)
An attorney claims that more than 25% of all lawyers advertise. To test this, we take random sample of 200 and count how many had used some form of advertising.

4. What would the null and alternative hypotheses be for our resulting hypothesis test?

5. If we find a test statistic of 2.12, which of the following is the corresponding p-value for our test?
A. 0.0340   B. 0.9830   C. 0.0170   D. 0.0680   E. 1.9660

6. When would our results be considered statistically significant?
A. When our p-value is greater than our significance level and we reject the null.
B. When our p-value is less than our significance level and we reject the null.
C. When our p-value is greater than our significance level and we fail to reject the null.
D. When our p-value is less than our significance level and we fail to reject the null.

7. If $\alpha = 0.01$, which of the following is the correct decision?
A. There is enough evidence to support the attorney's claim.
B. There is not enough evidence to support the attorney's claim.
C. There is enough evidence to refute the attorney's claim.
D. There is not enough evidence to refute the attorney's claim.

8. Suppose an airline interested in whether the proportion of on-time flights has increased. If data is collected and a hypothesis test is run, which of the following correctly describes a Type I error?
   A. To conclude the proportion of on-time flights has increased when it actually has.
   B. To conclude the proportion of on-time flights has increased when it actually hasn't.
   C. To conclude the proportion of on-time flights has not increased when it actually has.
   D. To conclude the proportion of on-time flights has not increased when it actually hasn't.

9. In order to estimate the mean cost of a hotel room in the Chicago area a random sample of 55 standard hotel rooms is taken. A 96% confidence interval was calculated to be ($170.77, $197.58). A Chicago tourist site states that the average hotel cost is $160. The confidence interval is used to test the hypotheses. Based on the confidence interval what would your decision be?
   A. Reject the null hypothesis: the null value does not fall within the confidence interval.
   B. Fail to reject the null hypothesis: the null value does not fall within the confidence interval.
   C. Reject the null hypothesis: the null value falls within the confidence interval.
   D. Fail to reject the null hypothesis: the null value falls within the confidence interval.

10. A researcher takes 100 different samples asking if people are in favor of a border wall and creates a 95% confidence interval for each sample. How many of those 100 resulting confidence intervals would you expect to contain the true proportion of people who are in favor of a border wall?
    A. 100   B. 95   C. 90   D. 10   E. 5

11. (6 points) A researcher wishes to estimate, with 90% confidence, the proportion of people who did not have a land line phone. If the researcher wishes to be accurate within 2% of the true proportion, how many people will she have to sample?

12. Which of the following is a **true** statement regarding the comparison of t-distributions to the standard normal distribution?
    A. The normal distribution is symmetrical whereas the t- distributions are slightly skewed.
    B. As the degrees of freedom increases, the t-distribution approaches the standard normal curve.
    C. The shape of the standard normal distribution changes as the sample size increases, but the shape of the t-distribution does not change.
    D. The total area under the t-distribution is larger than the total area under the standard normal distribution

13. A study was conducted to estimate the true proportion of Chuck Norris' round-house kicks that hit their intended target. A 95% confidence interval for the true proportion of Chuck Norris' kicks that land on their target was calculated to be (0.7788, 0.8212). What is the point estimate for the proportion of round-house kicks by Chuck Norris that hit their intended target?

Use the following information to answer questions 14 & 15
Diet Guide magazine claims that juice fasting for a week is an excellent way to lose weight. A sample of 20 individuals has a mean weight loss of 10.3 pounds, with a standard deviation of 4.8 pounds. It is known that weight loss follows a normal distribution.

14. (6 points) Create a 95% confidence interval for the true mean weight loss from juice fasting for a week.

15. Interpret the interval you found above.

16. A 95% confidence interval is computed to estimate the population proportion of adults in the U.S. who have had their identity stolen. Which of the following values will **definitely** be within the limits of this confidence interval?
    A. The sample proportion.
    B. The population proportion.
    C. The standard error.
    D. The critical value.

17. If you increase the sample size, what will happen to the sampling distribution of a sample mean?
    A. The mean and standard deviation would increase.
    B. The mean would stay the same and the standard deviation would increase.
    C. The mean and standard deviation would decrease.
    D. The mean would stay the same and the standard deviation would decrease.
    E. The mean and standard deviation will stay the same.

True or False (2 points each)
18. In a hypothesis test, the alternative hypothesis is what we are trying to prove.

19. If we get statistically significant results, it is possible that we may have committed a Type II error.

20. The Central Limit Theorem states that for any non-normal distribution, the sample means from that distribution will be approximately normally distributed, if the sample size is large enough.

21. (4 points) ** The p-value for a hypothesis test can be broadly defined as the probability of…
    A. The null hypothesis being true.
    B. The alternative hypothesis being false.
    C. Observing a sample as or more extreme than what was observed given that the null hypothesis is true.
    D. Making a Type I error.

22. (4 points) ** Suppose you are doing testing the following hypothesis to see if two groups had the same standard deviation or not.
$$H_0: \sigma_1 = \sigma_2$$
$$H_A: \sigma_1 \neq \sigma_2$$
The test reveals a p-value of 0.4523. What conclusion can you draw?

A

Quick survey:

How many hours did you study for the exam?

Predict your exam score out of 100:

What was the last math class you took at Mesa?

Goldilocks: I thought this exam was…

  Too easy / Just right / Too hard   Too short / Just right / Too long

Statistics 119 Formulas for the Semester

Probability Formulas:

$$P(A \ or \ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \ and \ B) = P(A \cap B) = P(A \mid B) \bullet P(B) \qquad\qquad P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Mean and standard deviation of a discrete random variable

$$E(X) = \mu = \sum xp(x) \qquad\qquad \sigma = \sqrt{\left(\sum (x - \mu)^2 \, p(x) \right)} = \sqrt{\sum x^2 p(x) - \mu^2}$$

Binomial Probability Function:

$$P(X = k) = {}_nC_k \ p^k q^{n-k}$$

$$Mean = \mu_x = np$$

$$Standard \ \ Deviation = \sigma_x = \sqrt{npq} \qquad\qquad z^* = \frac{x - np}{\sqrt{npq}} = \frac{x - \mu_x}{\sigma_x}$$

Sampling Distribution of a Sample Proportion:

$$Mean = \mu_{\hat{p}} = p$$

$$Standard \ Deviation = \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{pq}{n}} \qquad\qquad Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

Normal Distribution:

Direct calculation: $z = \dfrac{x - \mu}{\sigma}$ *Inverse calculation:* $x = z(\sigma) + \mu$

Mean and standard deviation of sample mean: $\mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

Direct calculation: $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ *Inverse calculation:* $\bar{x} = z\left(\dfrac{\sigma}{\sqrt{n}}\right) + \mu$

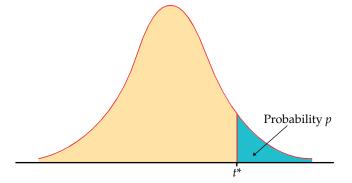|	Hypothesis Test	Confidence Interval	Sample Size
For proportions	$z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$	$\hat{p} \pm Z^* \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$	$n = \dfrac{(Z^*)^2 \, \hat{p}\hat{q}}{(ME)^2}$
For means (σ known)	$Z = \dfrac{\bar{x} - \mu}{\left(\sigma / \sqrt{n}\right)}$	$\bar{x} \pm Z^*\left(\dfrac{\sigma}{\sqrt{n}}\right)$	$n = \left(\dfrac{Z^* \sigma}{ME}\right)^2$
For means (σ unknown)	$t = \dfrac{\bar{x} - \mu}{\left(S / \sqrt{n}\right)}$	$\bar{x} \pm t^*\left(\dfrac{S}{\sqrt{n}}\right)$	$n = \left(\dfrac{(t^*)(s)}{ME}\right)^2$

Table entry for p and C is the critical value t* with probability p lying to its right and probability C lying between −t* and t*.

Probability p

t*

TABLE D

t distribution critical values

df	|	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
							Upper-tail probability p						
1		1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2		0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3		0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4		0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5		0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6		0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7		0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8		0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9		0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10		0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11		0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12		0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13		0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14		0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15		0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16		0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17		0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18		0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19		0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20		0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21		0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22		0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23		0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24		0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25		0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26		0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27		0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28		0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29		0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30		0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40		0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50		0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60		0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80		0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100		0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000		0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*		0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
		50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
							Confidence level C						

Table entry for z is
the area under the
standard normal curve
to the left of z.

Probability

z

TABLE A Standard normal probabilities

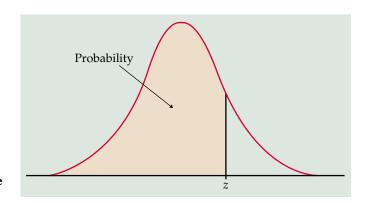| z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard normal curve to the left of z.

TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998