Name	Partner	Date

Lab 0: Measurement

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I. INTRODUCTION

Any time a measurement is made there is an associated, inherent uncertainty. This uncertainty is a function of both the device used to perform the measurement and experimenter/environment. If repeated measurements have been taken, statistical analysis may be used to determine the uncertainty in the measurement process. In this lab you will learn how to use statistics to determine the uncertainty for a repeated measurement. There will be some basic rules associated with the reporting of the uncertainty.

Rules for Uncertainties:

1. <u>The uncertainty should only have one significant digit.</u>
   For example, when the standard deviation of the measurements is calculated you might obtain a value such as 0.03567 cm, this should be reported as the uncertainty in your measurement as 0.04 cm. Since, the first significant digit is the three, but the next digit is a five so then you would round the three up to a four.
2. <u>Always round your reported result.</u>
   This will be the average of the repeated measurements, to the same place as the place in which the one significant digit in the uncertainty occurs. For example, if your average were 23.672 cm then you would report your measurement as 23.67 cm, which is rounded to the hundredths place. Otherwise, it appears that your result is more (or less) accurate than indicated by the uncertainty.
3. <u>Both your measurement and your uncertainty must have units and they really should be reported with the same units.</u>
   For example, your final result would be reported to be 23.67 cm ± 0.04 cm or (23.67 ± 0.04) cm. Also if your result is supposed to be reported in scientific notation, then they should both have the same power of ten based upon the result and not the uncertainty. You will move the decimal place the same way in the uncertainty as you did in the result and usually the power of ten is then factored out of the result as a common factor follows: $(2.367 \pm 0.004) * 10^1$ cm.

Lab objectives:

1. Learn about measurement techniques and data collection and processing with a specific emphasis on error analysis through a series of readings and exercises.

2. Apply knowledge gained in the first part of the lab by performing two different types of measurements and achieving calculated results.

II. AVERAGING, ERRORS & UNCERTAINTY EXERCIZES

Types of Error

There are three types of limitations to measurements:

1. **Instrumental limitations**
   Any measuring device can only be used to measure to with a certain degree of fineness. Our measurements are no better than the instruments we use to make them. *These should be included in your recorded and calculated values!*

2. **Systematic errors and blunders**
   These are caused by a mistake, **which does not change** during the measurement. For example, if the platform balance you used to weigh something was not correctly set to zero with no weight on the pan, all your subsequent measurements of mass would be too large. Systematic errors do not enter into the uncertainty. They are either identified and eliminated or lurk in the background producing a shift from the true value. *If you identify these they should be either calculated out of your results or, if that is not possible, the experiment needs to be repeated!*

3. **Random errors**
   These arise from unnoticed variations in measurement technique, tiny changes in the experimental environment, etc. Random variations affect precision. Truly random effects average out if the results of a large number of trials are combined.

Precision vs. Accuracy

A precise measurement is one where independent measurements of the same quantity closely cluster about a single value that may or may not be the correct value.

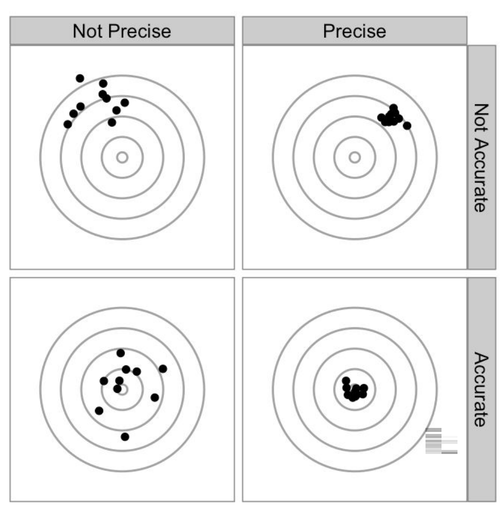
An accurate measurement is one where independent measurements cluster about the true value of the measured quantity.

> Systematic errors are not random and therefore can never cancel out. They affect the accuracy but not the precision of a measurement.

A. **Low-precision, Low-accuracy:**
   The average (the X) is not close to the center

B. **Low-precision, High-accuracy:**
   The average is close to the true value

C. **High-precision, Low-accuracy:**
   The average is not close to the true value



Writing Experimental Numbers

Uncertainty of Measurements

Errors are quantified by associating an uncertainty with each measurement. For example, the best estimate of a length ***L*** is 2.59 cm, but due to uncertainty, the length might be as small as 2.57 cm or as large as 2.61 cm. ***L*** can be expressed with its uncertainty in two different ways:

A. **Absolute Uncertainty**
   Expressed in the units of the measured quantity: ***L*** = **2.59** ± **0.02** cm

B. **Percentage Uncertainty**
   Expressed as a percentage which is independent of the units
   Above, since 0.02/2.59 ≈ 1% we would write ***L*** = **2.59** ± **1**%

Significant Figures

Experimental numbers must be written in a way consistent with the precision to which they are known. In this context one speaks of **significant figures** or digits that have physical meaning.

1. All definite digits and the first doubtful digit are considered significant.

2. Leading zeros are **not** significant figures.
   Example: ***L*** = **2.31** cm has 3 significant figures. For ***L*** = **0.0231** m, the zeros serve to move the decimal point to the correct position. Leading zeros are not significant figures.

3. Trailing zeros are significant figures: they indicate the number's precision.

4. One significant figure should be used to report the uncertainty or occasionally two, especially if the second figure is a five.

Rounding Numbers

To keep the correct number of significant figures, numbers must be rounded off. The discarded digit is called the **remainde**r. There are three rules for rounding:

Rule 1: If the remainder is less than 5, drop the last digit.
Rounding to one decimal place: 5.346 → 5.3

Rule 2: If the remainder is greater than 5, increase the final digit by 1.
Rounding to one decimal place: 5.798 → 5.8

Rule 3: If the remainder is exactly 5 then round the last digit to the closest even number. This is to prevent rounding bias. Remainders from 1 to 5 are rounded down half the time and remainders from 6 to 10 are rounded up the other half.
Rounding to one decimal place: 3.55 → 3.6, also 3.65 → 3.6

Examples

The period of a pendulum is given by $T = 2\pi\sqrt{l/g}$.

Here, $l = 0.24\text{m}$ is the pendulum length and $g = 9.81\text{m/s}^2$ is the acceleration due to gravity.

✗**WRONG:** $T = 0.983269235922\text{s}$

✓**RIGHT:** $T = 0.98\text{s}$

Your calculator may report the first number, but there is no way you know T to that level of precision. When no uncertainties are given, report your value with the same number of significant figures as the value with the smallest number of significant figures.

The mass of an object was found to be 3.56g with an uncertainty of 0.032g.

✗**WRONG:** $m = 3.56 \pm 0.032\text{g}$

✓**RIGHT:** $m = 3.56 \pm 0.03\text{g}$

The first way is wrong because the uncertainty should be reported with one significant figure

The length of an object was found to be 2.593cm with an uncertainty of 0.03cm.

✗**WRONG:** $L = 2.593 \pm 0.03\text{cm}$

✓**RIGHT:** $L = 2.59 \pm 0.03\text{cm}$

The first way is wrong because it is impossible for the third decimal point to be meaningful.

The velocity was found to be 2.45m/s with an uncertainty of 0.6m/s.

✗**WRONG:** $v = 2.5 \pm 0.6\text{m/s}$

✓**RIGHT:** $v = 2.4 \pm 0.6\text{m/s}$

The first way is wrong because the first discarded digit is a 5. In this case, the final digit is rounded to the closest even number (i.e. 4)

The distance was found to be 45600m with an uncertainty around 1m

✗**WRONG:** $d = 45600\text{m}$

✓**RIGHT:** $d = 4.5600 \times 10^4\text{m}$

The first way is wrong because it tells us nothing about the uncertainty. Using scientific notation emphasizes that we know the distance to within 1m.

Statistical Analysis of Small Data Sets

Repeated measurements allow you to not only obtain a better idea of the actual value, but also enable you to characterize the uncertainty of your measurement. Below are a number of quantities that are very useful in data analysis. The value obtained from a particular measurement is **x**. The measurement is repeated N times. Oftentimes in lab **N** is small, usually no more than 5 to 10. In this case we use the formulae below:

Mean (x_{avg})	The average of all values of x (the "best" value of x)	$x_{\text{avg}} = \frac{x_1 + x_2 + \cdots + x_N}{N}$
Range (R)	The "spread" of the data set. This is the difference between the maximum and minimum values of x	$R = x_{\text{max}} - x_{\text{min}}$
Uncertainty in a measurement (Δx)	Uncertainty in a single measurement of x. You determine this uncertainty by making multiple measurements. You know from your data that x lies somewhere between x_{max} and x_{min}.	$\Delta x = \frac{R}{2} = \frac{x_{\text{max}} - x_{\text{min}}}{2}$
Uncertainty in the Mean (Δx_{avg})	Uncertainty in the mean value of x. The actual value of x will be somewhere in a neighborhood around x_{avg}. This neighborhood of values is the uncertainty in the mean.	$\Delta x_{\text{avg}} = \frac{\Delta x}{\sqrt{N}} = \frac{R}{2\sqrt{N}}$
Measured Value (x_{m})	The final reported value of a measurement of x contains both the average value and the uncertainty in the mean.	$x_{\text{m}} = x_{\text{avg}} \pm \Delta x_{\text{avg}}$

The average value becomes **more and more precise** as the number of measurements N increases. Although the uncertainty of any single measurement is always Δx, the **uncertainty in the mean** $\Delta \mathbf{x}_{\mathbf{avg}}$ becomes smaller (by a factor of $\frac{1}{\sqrt{N}}$) as more measurements are made.

Example

You measure the length of an object five times.
You perform these measurements twice and obtain the two data sets below.

Measurement	Data Set 1 (cm	Data Set 2 (cm)
x_1	72	80
x_2	77	81
x_3	82	81
x_4	86	81
x_5	88	82

Quantity	Data Set 1 (cm)	Data Set 2 (cm)
$\boldsymbol{x_{avg}}$	**81**	**81**
$\boldsymbol{R}$	**16**	**2**
$\boldsymbol{\Delta x}$	**8**	**1**
$\boldsymbol{\Delta x_{avg}}$	**4**	**0.4**

For Data Set 1, to find the best value, you calculate the mean (i.e. average value):

$$x_{avg} = \frac{72\text{cm} + 77\text{cm} + 82\text{cm} + 86\text{cm} + 88\text{cm}}{5} = 81\text{cm}$$

The range, uncertainty and uncertainty in the mean for Data Set 1 are then:

$$R = 88\text{cm} - 72\text{cm} = 16\text{cm}$$

$$\Delta x = \frac{R}{2} = 8\text{cm}$$

$$\Delta x_{avg} = \frac{R}{2\sqrt{5}} \approx 4\text{cm}$$

Data Set 2 yields the same average but has a much smaller range.

We report the measured lengths x_{m} as:

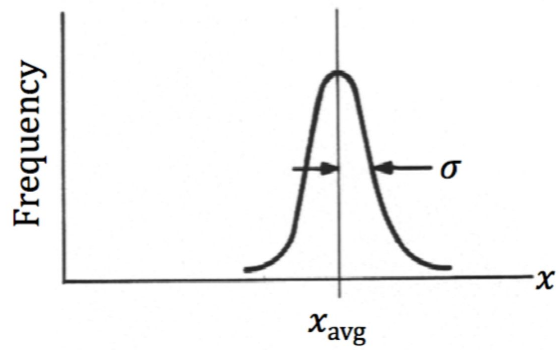
Data Set 1: $\boldsymbol{x_{\mathrm{m}} = 81 \pm 4\text{cm}}$

Data Set 2: $\boldsymbol{x_{\mathrm{m}} = 81.0 \pm 0.4\text{cm}}$

Notice that for Data Set 2, Δx_{avg} is so small we had to add another significant figure to x_{m}.

Statistical Analysis of Large Data Sets

If only random errors affect a measurement, it can be shown mathematically that in the limit of an **infinite** number of measurements ($N \to \infty$), the distribution of values follows a **normal distribution** (i.e. the bell curve on the right). This distribution has a peak at the mean value x_{avg} and a width given by the standard deviation σ.



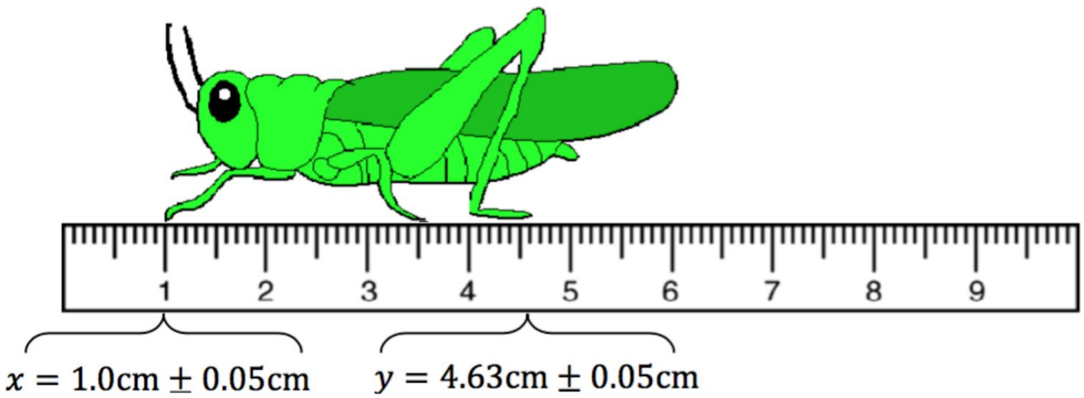
Obviously, we never take an infinite number of measurements. However, for a large number of measurements, say, ~10 - 10^2 or more, measurements may be approximately normally distributed. In that event we use the formulae below:

Mean (x_{avg})	The average of all values of x (the "best" value of x). This is the same as for small data sets.	$x_{\text{avg}} = \frac{\sum_{i=1}^{N} x_i}{N}$
Uncertainty in a measurement (Δx)	Uncertainty in a single measurement of x. The vast majority of your data lies in the range $x_{\text{avg}} \pm \sigma$	$\Delta x = \sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - x_{\text{avg}})^2}{N}}$
Uncertainty in the Mean (Δx_{avg})	Uncertainty in the mean value of x. The actual value of x will be somewhere in a neighborhood around x_{avg}. This neighborhood of values is the uncertainty in the mean.	$\Delta x_{\text{avg}} = \frac{\sigma}{\sqrt{N}}$
Measured Value (x_{m})	The final reported value of a measurement of x contains both the average value and the uncertainty in the mean.	$x_{\text{m}} = x_{\text{avg}} \pm \Delta x_{\text{avg}}$

Most of the time we will be using the formulae for small data sets. However, occasionally we perform experiments with enough data to compute a meaningful standard deviation. In those cases we can take advantage of software that has programmed algorithms for computing x_{avg} and σ.

Propagation of Uncertainties

Oftentimes we combine multiple values, each of which has an uncertainty, into a single equation. In fact, we do this every time we measure something with a ruler. Take, for example, measuring the distance from a grasshopper's front legs to his hind legs. For rulers, we will assume that the uncertainty in all measurements is one-half of the smallest spacing.



The measured distance is $d_m = d \pm \Delta d$ where d = 4.63 cm -1.0 cm = 3.63 cm. What is the uncertainty in d_m? You might think that it is the sum of the uncertainties in x and y (i.e. $\Delta d = \Delta x + \Delta y = 0.1$ cm). However, statistics tells us that if the uncertainties are independent of one another, the uncertainty in a sum or difference of two numbers is obtained by $\Delta d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = 0.07cm$. The way these uncertainties combine depends on how the measured quantity is related to each value. Rules for how uncertainties propagate are given below.

Addition/Subtraction	$z = x \pm y$	$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
Multiplication	$z = xy$	$\Delta z = \lvert xy \rvert \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
Division	$z = \frac{x}{y}$	$\Delta z = \left\lvert \frac{x}{y} \right\rvert \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
Power	$z = x^n$	$\Delta z = \lvert n \rvert x^{n-1} \Delta x$
Multiplication by a Constant	$z = cx$	$\Delta z = \lvert c \rvert \Delta x$
Function	$z = f(x, y)$	$\Delta z = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\Delta y)^2}$

Examples

Addition

The sides of a fence are measured with a tape measure to be 124.2cm, 222.5cm, 151.1cm and 164.2cm. Each measurement has an uncertainty of 0.07cm. Calculate the measured perimeter P_{m} including its uncertainty.

$$P = 124.2\mathrm{cm} + 222.5\mathrm{cm} + 151.1\mathrm{cm} + 164.2\mathrm{cm} = 662.0\mathrm{cm}$$

$$\Delta P = \sqrt{(0.07\mathrm{cm})^2 + (0.07\mathrm{cm})^2 + (0.07\mathrm{cm})^2 + (0.07\mathrm{cm})^2} = 0.14\mathrm{cm}$$

$$\boxed{P_{\mathrm{m}} = 662.0 \pm 0.1\mathrm{cm}}$$

Multiplication

The sides of a rectangle are measured to be 15.3cm and 9.6cm. Each length has an uncertainty of 0.07cm. Calculate the measured area of the rectangle A_{m} including its uncertainty.

$$A = 15.3\mathrm{cm} \times 9.6\mathrm{cm} = 146.88cm^2$$

$$\Delta A = 15.3\mathrm{cm} \times 9.6\mathrm{cm}\sqrt{\left(\frac{0.07}{15.3}\right)^2 + \left(\frac{0.07}{9.6}\right)^2} = 1.3\mathrm{cm}^2$$

$$\boxed{A_{\mathrm{m}} = 147 \pm 1\mathrm{cm}^2}$$

Power/Multiplication by Constant

A ball drops from rest from an unknown height h. The time it takes for the ball to hit the ground is measured to be $t = 1.3 \pm 0.2\mathrm{s}$. The height is related to this time by the equation $h = \frac{1}{2}gt^2$ where $g = 9.81\mathrm{m/s^2}$. Assume that the value for g carries no uncertainty and calculate the height h including its uncertainty.

$$h = \frac{1}{2}(9.8\mathrm{m/s^2})(1.3\mathrm{s})^2 \approx 8.281\mathrm{m}$$

$$\Delta h = \frac{1}{2}(9.8\mathrm{m/s^2})(2 \times 1.3\mathrm{s} \times 0.2\mathrm{s}) \approx 2.5\mathrm{m}$$

$$\boxed{h_{\mathrm{m}} = 8 \pm 3\mathrm{m}}$$

Problems

1. State the number of significant figures in each of the following numbers and explain your answer.
   a. 37.60

   b. 0.0130

   c. 13000

   d. 1.3400

2. Perform the indicated operations to the correct number of significant figures using the rules for significant figures.
   a. $37.60 * 1.23 =$

   b. $\frac{9.975}{6.7} =$

   c. $3.765 + 1.2 + 37.21 =$

3-7. Three students named Amer, Barb, and Cal make measurements (in m) of the length of a table using a meter stick. Each student's measurements are tabulated in the table below along with the mean, the standard deviation from the mean, and the standard error of the measurements. Note that in each case only one significant figure is kept in the average uncertainty (Δx_{avg}), and this determines the number of significant figures in the mean. The actual length of the table is determined by very sophisticated laser measurement techniques to be ***1.4715 m***.

	Amer	**Barb**	**Cal**
$\mathbf{x_1}$	1.4717	1.4753	1.4719
$\mathbf{x_2}$	1.4711	1.4759	1.4723
$\mathbf{x_3}$	1.4722	1.4756	1.4727
$\mathbf{x_4}$	1.4715	1.4749	1.4705
$\mathbf{x_{avg}}$	1.4716	1.4754	1.4719
R	0.0011	0.0010	0.0022
$\mathbf{\Delta x}$	0.0006	0.0005	0.0011
$\mathbf{\Delta x_{avg}}$	0.0003	0.0002	0.0006

3. Write the measured lengths for each of the students.

4. State how one determines the accuracy of a measurement. Apply your idea to the measurements of the three students above and state which of the students has the most accurate measurement. Why is that your conclusion?

5. Calculate the mean, range, uncertainty in the measurement, and uncertainty in the mean for Amer's measurements of length. Confirm that your calculated values are the same as those in the table. Show your calculations explicitly.

	Amer
x_{avg}	
R	
Δx	
Δx_{avg}	

6. State the characteristics of data that indicate a systematic error. Do any of the three students have data that suggest the possibility of a systematic error? If so, state which student it is, and state how the data indicate your conclusion.

7. Which student has the best measurement considering both accuracy and precision? State clearly what the characteristics are of the student's data on which your answer is based.

III. COLLECTING & ANALYZING DATA

Materials/Equipment

Plastic ruler and meter stick (or comparable measuring devices), table in your home

Measurement 1: Lab Table

A. Procedure

Measure the length and width of a table 10 times, first with the meter stick and then with the plastic ruler. When you are finished, average the values to get a better measure of the piece's true length. Make an "eyeball" estimate of your uncertainties. Keep in mind that the better and more precise/accurate your technique (as described below) the better your grade will be.

B. Data Collection

	Meter Stick		**Plastic Ruler**	
Trial	**L** (m)	**W** (m)	**L** (m)	**W** (m)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
Avg				
R				
Δx				
Δx_{avg}				

C. Data Analysis

1. Describe (in full, grammatically correct sentences) your measurement technique for the measuring length and width with
    a. the meters stick
    b. the ruler
2. Why do we need to take so many measurements?
3. Write out the length and width of the table including uncertainty.
    a. For the meters stick
    b. for the ruler
4. Explain the possible sources of error in this measurement. How did you come up with the number of significant figures you used?
5. How well did your measurements with the meter stick agree with those done with the smaller, plastic ruler? If there was a disagreement, what kind of error was it? Random or systematic? What caused this error?

D. Calculations

1. Calculate area of table including the propagation of uncertainty (explained in the previous section).

2. Calculate perimeter of table including propagation of uncertainty (explained in the previous section)

IV. QUESTIONS/CONCLUSIONS

1. Which of your measurements was the most uncertain? Why?

2. Which of your measurements was the least uncertain? Why?

3. Which measurements, if any, suffered from systematic error? Explain.