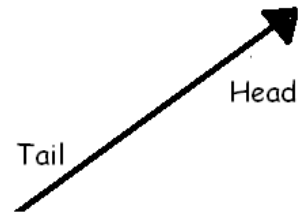


Virtual Lab – Vectors & Vector Operations

Setup

1. Make sure your calculator is set to degrees and not radians.
2. Once you get to the **Vector Addition** page, you can press the play button, or download the application. Note that you need Java to play the simulation if downloaded.
3. Click on the button that says “Explore 2D”.
4. In the basket at the right, you can drag out a vector arrow. If you ever want to get rid of a vector, drag it off the grid. If you want to start over, click the eraser button.
5. You can adjust the direction and length of the arrow by click-dragging the arrow head. Play with this until you are comfortable.
6. Check all the boxes at the top right except for “sum.” (these will show grid, vector length, and angles)



Part A: 3-4-5 Triangle

7. Drag out a vector, and move it until the tail is located at the origin. Click on the head of the vector, and drag it until it is completely horizontal, points to the right, and has a magnitude (\vec{a}) of 20.
8. Look at the chart at the top of the page. Here is an explanation of what each number represents:
 - a. \vec{a} represents the length of the arrow. This is usually called the **magnitude** of the vector.
 - b. θ represents the direction the arrow points. This is simply called the **direction** of the vector. The magnitude AND direction will completely define a vector.
 - c. a_x is called the **X-component** of the vector. This is the length of the vector in the X-direction only.
 - d. a_y is called the **Y-component** of the vector. This is the length of the vector in the Y-direction only.
9. For the first vector you dragged out, fill in the chart at right.

\vec{a}	θ	a_x	a_y

10. Now, drag out a second vector \vec{b} and place its tail at the head of the first, as shown at right. Adjust this second vector until it points vertically upward and has a length of 15. Fill in the table for this vector here:


\vec{b}	θ	b_x	b_y



11. If you were to walk this path, at the end you would be 25 units away from the origin. You can show this by clicking the button that says **Sum**. A new dark blue vector \vec{s} should pop up. This represents the vector sum, or **resultant**, of the first two arrows.

12. Drag this vector over so that the tail is at the origin, and use it to form the hypotenuse of a right triangle. Notice that the head of this vector ends exactly where the second vector ends. Click on the dark blue vector and fill in the chart for this vector here:


s	θ	s_x	s_y

13.  Compare the s_x and s_y values to the values from questions #9 and #10. What do you notice about these values?


Part B: Single Vector, Magnitude 50

14. Hit the **Eraser** button to erase the screen. Next, create a new \vec{a} vector with an a_x of 20 and an a_y of 15.
15. Fill in the chart for this vector here:

\vec{a}	θ	a_x	a_y




15.  Compare the chart values of this vector to those of the dark blue \vec{S} vector from #12. How do these values compare?



16. Next, click the  button on the “Components” menu. This is a way to visualize any vector as a sum of horizontal and vertical components.

17. Adjust this vector until it has an a_x value of 30 and an a_y value of 15. Fill in the chart for this vector:

\vec{a}	θ	a_x	a_y

18.  Has the **magnitude** of this vector changed, compared #14? If so, how?
19.  Has the **direction** (that is, θ) of this vector changed, compared to #14? If so, how?
20.  Figure out a way to adjust the magnitude and direction of this vector until it has a magnitude of 25, just like before, but points in a *different* direction from the first two tries. Fill in the chart for this vector.

\vec{a}	θ	a_x	a_y


21. Looking at this vector, it is easy to imagine a right triangle, made from a_x , a_y and $|\vec{a}|$. In this case, $|\vec{a}|$ would be the hypotenuse, and a_x & a_y would be the legs.

a. Show, using the Pythagorean Theorem, that $|\vec{a}|^2 = a_x^2 + a_y^2$.

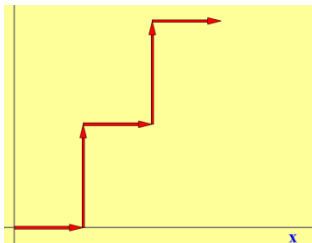
b. Show, using SOHCAHTOA, that $a_x = |\vec{a}| \cos \theta$.

c. Show, using SOHCAHTOA, that $a_y = |\vec{a}| \sin \theta$.

22. Clear All. Imagine a vector with magnitude $|\vec{a}| = 28$ and angle $\theta = 45^\circ$.


- a.  Use SOHCAHTOA to determine the X- And Y- components (that is, find a_x and a_y).
- b. Check your answer by constructing this vector.

Part C – Several Vectors (Switch to “Lab” version of sim)





23. Click on the “Lab” icon at the bottom. Create 5 vectors of the same color, as shown at right. The length of each of the horizontal vectors should be 5, and the length of the vertical vectors should be 10.
24. Click on the “Show Sum” button. Fill in the chart for this resultant, or sum, vector.

\vec{s}	θ	s_x	s_y

25.  A useful way to keep track of vector sums is to create a chart. Complete the chart below, using the 5 vectors you’ve constructed, and then add the columns to get the sums.

Vector #	v_x	v_y
1	5	0
2		
3		
4		
5		
SUM		

26.  How do the v_x and v_y sums from the previous chart compare to the s_x and s_y values from question #24?
27.  Using the Pythagorean Theorem, determine the resultant $|\vec{s}|$ value, which is the hypotenuse of a triangle with s_x and s_y as its legs. Compare this number to the $|\vec{s}|$ value from #24.
28. **Clear All.** Construct the following 4 vectors and add them with the “head-to-tail” method:
- $\vec{v} = 10, \theta = 0^\circ$ (start this one at the origin)
 - $\vec{v} = 10, \theta = 90^\circ$
 - $\vec{v} = 10, \theta = 180^\circ$ (or -180°)
 - $\vec{v} = 10, \theta = 270^\circ$ (or -90°)

29. What is the sum (or resultant) of these vectors?

\vec{s}	θ	s_x	s_y

30. What is the sum of these vectors if the first vector is 20 units long rather than 10?

\vec{s}	θ	s_x	s_y

Extension Questions

1. A student, following instructions on her treasure map, starts at the origin and walks the following routes:

→18 meters North ($\theta = 90^\circ$)
→5 meters West ($\theta = 180^\circ$)
→9 meters South ($\theta = 270^\circ$ or -90°)
→17 meters East ($\theta = 0^\circ$)

- a. Fill in the chart below, which represents the horizontal and vertical components of the routes. Also determine the X and Y sums.

Vector #	v_x	v_y
1	0	18
2		
3		
4		
SUM		

- b. After the student has finished walking, what is her horizontal displacement? (v_x sum)
- c. What is her vertical displacement? (v_y sum)
- d. Using the Pythagorean Theorem, and your answers from (b) and (c), how far is she from the origin? (In other words, what is her resultant $|R|$?)
- e. Using SOHCAHTOA, what is her direction, as measured from the origin? (In other words, what is θ ?)
2. A model airplane is flying North with a velocity of 15 m/s. A strong wind is blowing East at 12 m/s.
- a. What is the airplane's resultant **speed** (magnitude of velocity vector)?
- b. What is the airplane's **heading** (direction of velocity vector)?